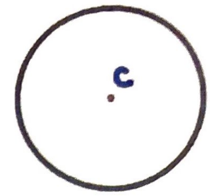
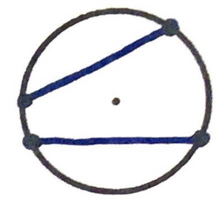
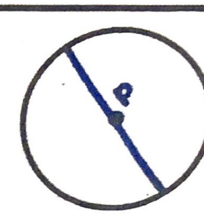
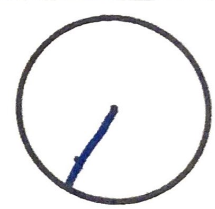
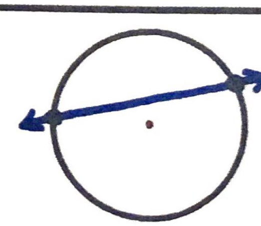
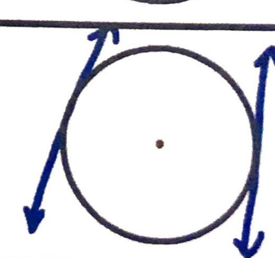
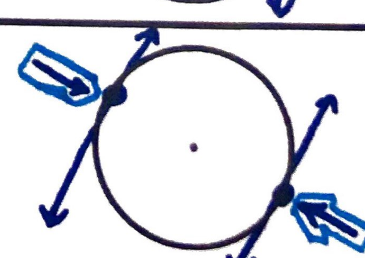
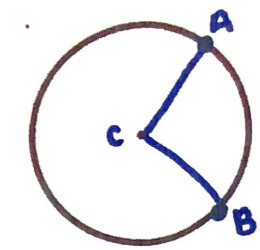
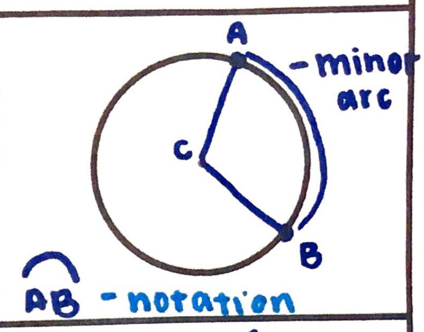
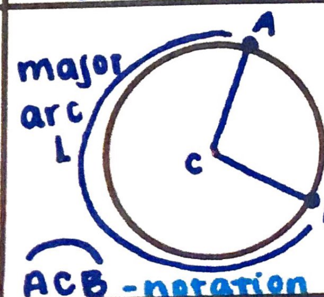
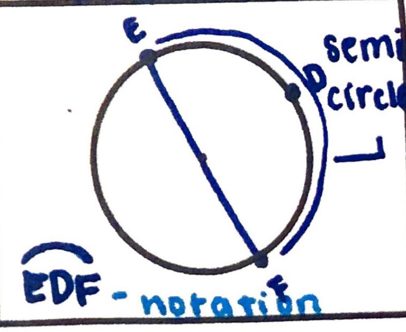
Unit 4A: Circles

1. vocab
2. ↓
3. Central Angles
4. Angles in Circles
5. Inscribed polygons
6. more central & inscribed
7. vertex outside/inside
8. Arclength/Sector Area

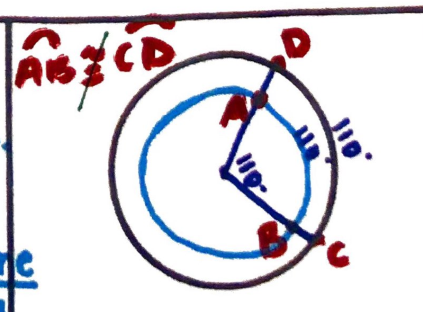
Circles Vocab

Circle	set of all points that are equidistant from a given point called the center of the circle.	
Chord	A segment that's endpoints are on the circle.	
Diameter	Distance across the circle through the center of the circle. AKA the longest chord.	
Radius	Distance from the center to the edge of the circle. -Radius = ½ diamiter -Diamiter = 2(radius)	
Secant	Intersects the circle at exactly <u>two</u> points.	
Tangent	a line that intersects at exactly <u>one</u> point.	
Point of Tangency	The point where the tangent intersects with the circle. -If you draw a radius from a tan. line, it ALWAYS forms a RA	

Central Angle	An angle whose vertex is at the center of the circle. - measure Arc = measure central Angle.	
Minor Arc	An arc that measures less than 180°.	
Major Arc	An arc that measures more than 180°	
Semicircle	An arch that equals 180°.	

You must remember:

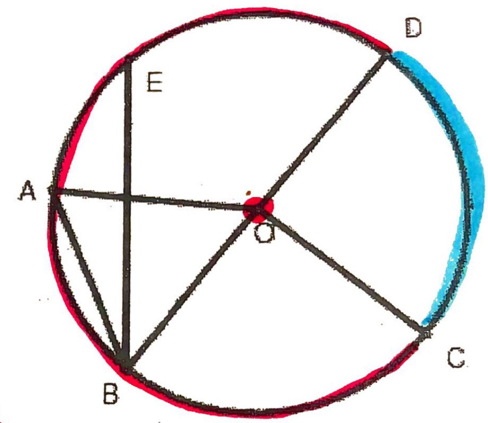
- A circle has 360° degrees
- A semicircle has 180 degrees
- Vertical Angles are congruent
- Linear Pairs are supplementary

Congruent Arcs	These have the same measure and MUST come from the same circle or of congruent circles. Arc length / unit of distance vs. Arc measure / in degrees	

Central Angles

Central Angles

1. Identify and name each of the following. Be sure to use the correct notation.



a. Two different central angles $\angle DOC$, $\angle COB$

2 letters b. A minor arc $\overset{\frown}{DC}$

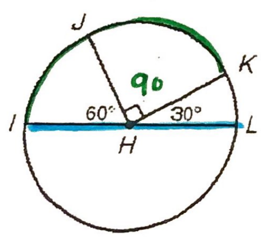
c. A major arc $\overset{\frown}{DBC}$

d. A semicircle $\overset{\frown}{BAD}$

e. Two different chords $\overline{AB}$, $\overline{EB}$

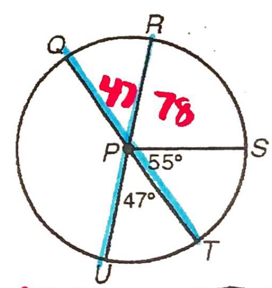
f. The central angle subtended by $\widehat{AD}$ $\angle DOA$

Find each measure.

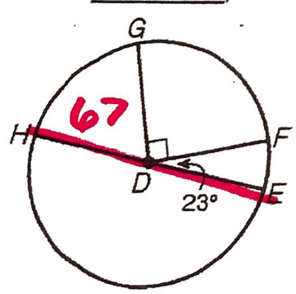


2. $m\widehat{LK}$ 30° , $m\widehat{IK}$ 150°

180-47-55 = 78



3. $m\widehat{QS}$ 125 $m\widehat{RQT}$ 227

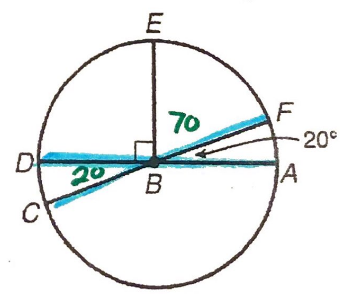


90+23

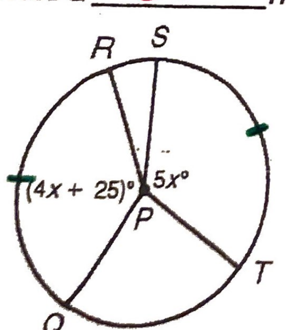
180-113

180+23 =

4. $m\widehat{HG}$ 67° $m\widehat{FEH}$ 203



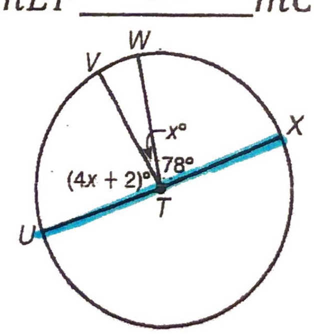
5. $m\widehat{EF}$ 70° $m\widehat{CEA}$ 200°



4x+25 = 5x

25 = x

6. $\angle QPR$ 125°



4x+2+x = 102

5x = 100

x = 20

7. $\angle UTW$ 102 , $m\widehat{UV}$ 82

vertex is center of circle

Central

mAngle = mArc

Angles in Circles

CENTRAL ANGLE—VERTEX IS THE center OF THE CIRCLE

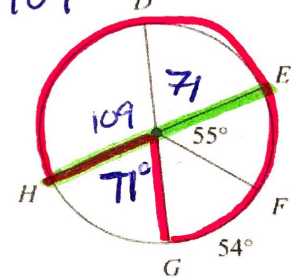
Measure of the Angle = Measure of the Arc $m\widehat{HEG}$

$$\begin{array}{r} 54 \\ +55 \\ \hline 109 \end{array}$$

$180 - 109 = 71$

$m\widehat{HEG} = 360 - 71$

$\boxed{m\widehat{HEG} = 289^\circ}$



Inscribed

$m\text{Angle} = \dfrac{m\text{Arc}}{2}$

INSCRIBED ANGLE—VERTEX IS ON THE CIRCUMFERENCE OF THE CIRCLE

Measure of the Angle = $\frac{1}{2}$ Measure of Intercepted Arc

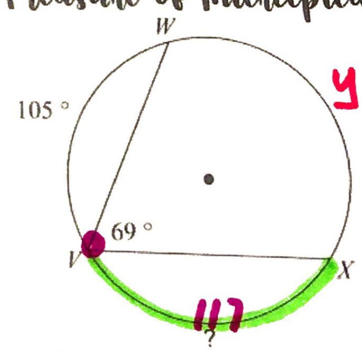
$2 \cdot 69 = y/2 \cdot 2$

$138^\circ = y$

$y = 138^\circ$

$? = 360 - 105 - 138$

$\boxed{? = 117^\circ}$



vertex is on the circle

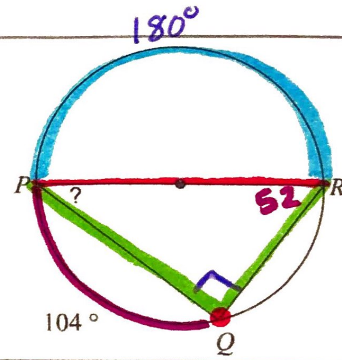
Inscribed POLYGONS

• Sometimes we have to break them down into their individual inscribed angles to solve problems.

TRIANGLE INSCRIBED IN A CIRCLE—IF THE DIAMETER OF THE CIRCLE IS A SIDE LENGTH OF A TRIANGLE INSCRIBED IN A CIRCLE, THEN IT IS THE Hypotenuse OF A RIGHT TRIANGLE.

$90-52=?$

$\boxed{38=?}$

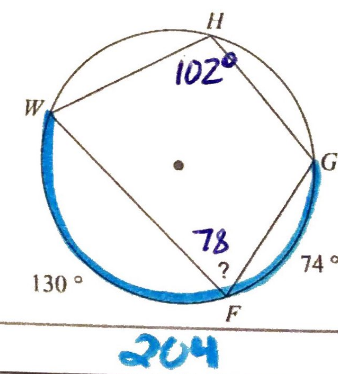


QUADRILATERAL INSCRIBED IN A CIRCLE:

THE OPPOSITE ANGLES ARE **Supplementary**

$180-102=?$

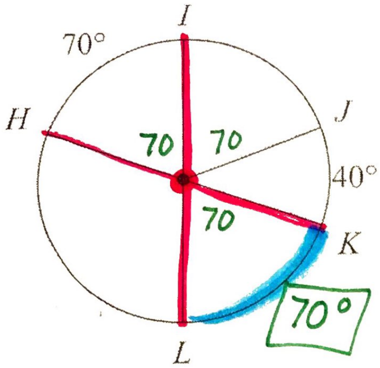
$78° = ?$



1 Central Angles

Find: $m\widehat{KL}$

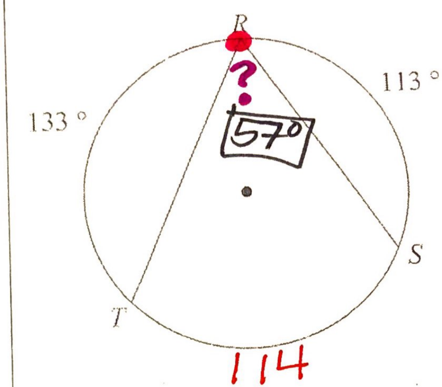
mangle = marc



$\boxed{70°}$

2 Inscribed Angle

Find ? mAngle = $\frac{\text{mArc}}{2}$



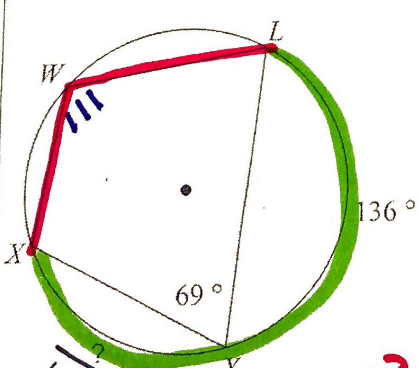
$\boxed{57°}$

133 + 113
246
360 − 246
114
$\frac{114}{2}$

3 Quadrilateral Inscribed

opposite angles supplementary

Find ?



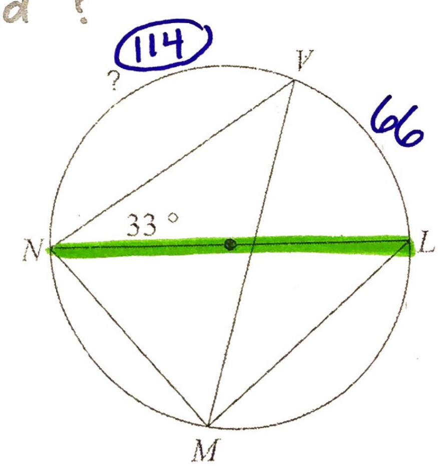
$\boxed{86°}$

222
− 136
86

Look for Diameters!

Find ?

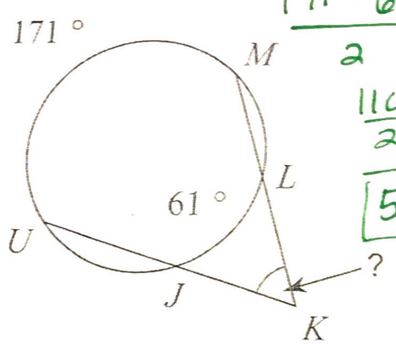
180
− 66



Vertex Outside & Inside

vertex outside

$\frac{LA - SA}{2} = \text{Angle}$



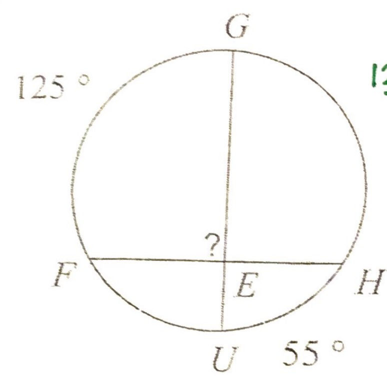
$\frac{171 - 61}{2} = ?$

$\frac{110}{2} = ?$

$\boxed{55 = ?}$

vertex inside, not center

$\text{angle} = \frac{\text{Arc} + \text{Arc}}{2}$



$\frac{125 + 55}{2} = ?$

$\frac{180}{2} = ?$

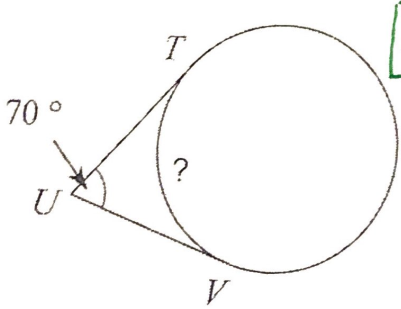
$90 = ?$

Special Case *TanTan*

$180 - SA = \text{angle}$

$180 - x = 70$

$\boxed{x = 110}$

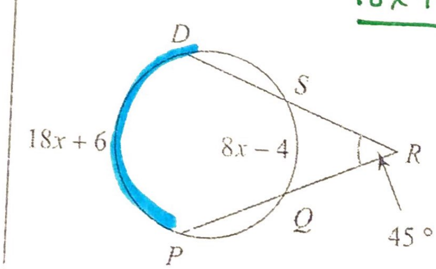


Vertex outside

$\frac{LA - SA}{2} = \text{angle}$

Find $m\overset{\frown}{DP}$

$\frac{18x + 6 - (8x - 4)}{2} = 45$



$18x + 6 - 8x + 4 = 90$

$10x + 10 = 90$

$10x = 80$

$\boxed{x = 8}$

$18(8) + 6 = \overset{\frown}{DP}$

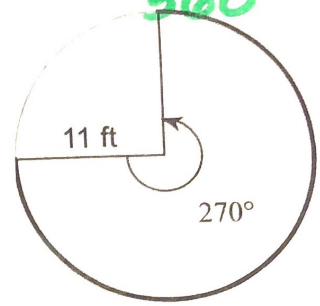
$\boxed{150 = \overset{\frown}{DP}}$

Arc Length and Sector Area

Arc Length—a portion of the total Circumfrence

Formula: $A\ell = \frac{\theta}{360}(2\pi r)$ or $A\ell = \frac{2\pi r\theta}{360}$

Example: Find the length of the arc indicated in bold



$r = 11 \quad \theta = 270$

$A\ell = \frac{270}{360}(22\pi)$

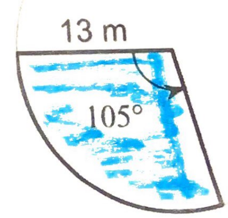
$A\ell = \frac{3}{4}\cdot(22\pi)$

$A\ell = \boxed{\frac{33\pi}{2}} \approx \boxed{51.84\text{ft}}$

Sector Area—a portion of the total Area

Formula: $A_s = \frac{\theta}{360}(\pi r^2)$ or $A_s = \frac{\pi r^2\theta}{360}$

Example: Find the area of the shaded region



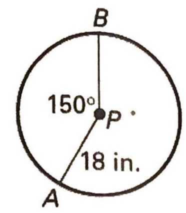
$A_s = \frac{105}{360}(169\pi)$

$A_s = \frac{7}{24}\cdot 169\pi$

$A_s = \boxed{\frac{1183\pi}{24}} \approx \boxed{154.85\text{ m}^2}$

Examples of Each

1. Find the arclength of arc AB.



$A\ell = \frac{150}{360}(36\pi)$

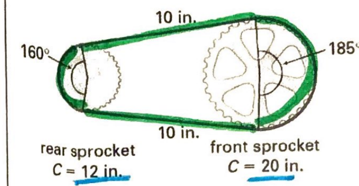
$= \frac{5}{12} \cdot 36\pi$

$\boxed{A\ell = 15\pi}$

$\boxed{\text{Approx}: 47.12\text{ in}}$

2. Application of Arc Length:

Bicycles The chain of a bicycle travels along the front and rear sprockets, as shown. The circumference of each sprocket is given.



Rear

$A\ell = \frac{160}{360}(12)$

$A\ell = 5.\overline{33}$

Front

$A\ell = \frac{185}{360}(20)$

$A\ell = 10.28$

a. About how long is the chain?

$5.33 + 10.28 + 20 =$

Chain: $\boxed{35.61\text{ in}}$

3. Sector Area: Find the area of the sector bounded by arc AB.

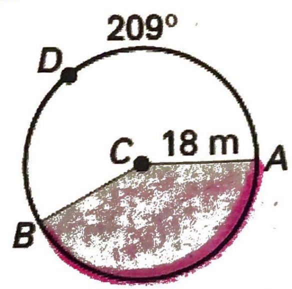
$\overset{\frown}{AB} = 360 - 209$

$\overset{\frown}{AB} = 151$

$A_s = \frac{151}{360}(324\pi)$

$= \frac{1359\pi}{10}\ m^2$

$\approx 426.94\ m^2$



4. Working Backwards with Sector Area

$A_s = \frac{\theta}{360}(\pi r^2)$

SUBSTITUTE!

Find the radius of $\odot H$.

$40.62 = \frac{98}{360}(\pi r^2)$

$\frac{180}{49} \cdot 40.62 = \frac{49}{180}(\pi r^2) \cdot \frac{180}{49}$

$\frac{149.22}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{47.5} = \sqrt{r^2}$

$\boxed{r = 6.89\text{ in}}$