Solving Quadratic Equations – Matching Name:

Solve the following by: greatest common factor, factoring, square roots, or completing the square. Then, match the equation to the answer(s) on the right.

H 1) $x^2 - 16x + 63 = 0$

B 2) $x^2 + 6x - 2 = 0$

A 3) $5x^2 = 45$

E 4) $x^2 - 2x - 14 = -4$

S 5) $4x^2 + 20x - 20 = 4$

D 6) $(x + 3)^2 + 2 = -10$

I 7) $2x^2 - 3x = 0$

K 8) $x^2 - 4x - 18 = -x$

Q 9) $x^2 + 14x - 30 = 8$

P 10) $3x^2 - 2x = 8$

A 11) $x^2 - 9 = 0$

G 12) $5x^2 + 9 = 134$

L 13) $x^2 - 8x + 3 = 0$

C 14) $2x^2 + x - 10 = 0$

J 15) $2(x - 3)^2 - 12 = 4$

R 16) $2x^2 + x - 10 = 5$

N 17) $x^2 - 8x - 33 = 0$

F 18) $x^2 - 4x - 12 = 0$

M 19) $x^2 - 10x - 8 = 0$

O 20) $2(x - 3)^2 = 8$

one answer will be used twice

a) $x = 3, x = -3$

b) $x = -3 \pm \sqrt{11}$

c) $x = 2, x = -\frac{5}{2}$

d) no real solution

e) $x = 1 \pm \sqrt{11}$

f) $x = -2, x = 6$

g) $x = \pm 5$

h) $x = 7, x = 9$

i) $x = 0, x = \frac{3}{2}$

j) $x = 3 \pm 2\sqrt{2}$

k) $x = -3, x = 6$

l) $x = 4 \pm \sqrt{13}$

m) $x = 5 \pm \sqrt{33}$

n) $x = -3, x = 11$

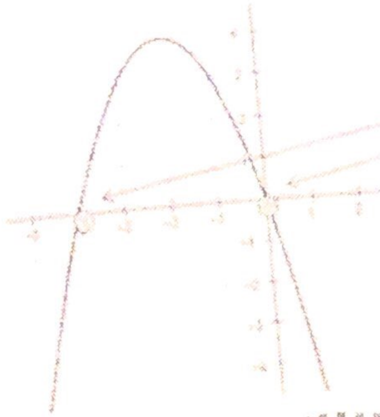
o) $x = 1, x = 5$

p) $x = 2, x = -\frac{4}{3}$

q) $x = -7 \pm \sqrt{87}$

r) $x = -3, x = \frac{5}{2}$

s) $x = -6, x = 1$



x-intercepts
= solutions
= roots
= zeros

(-4, 0) and (0, 0)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

Solving Quadratic Equations

x	f(x)
0	6
1	0
4	-6
6	0
7	6

(1,0) is a root or solution to the quadratic function $f(x) = x^2 - 7x + 6$.

Check your solution by substituting.
$0 = 1^2 - 7(1) + 6$
$0 = 1 - 7 + 6$
$0 = 0$

(6, 0) is a root or solution to the quadratic function $f(x) = x^2 - 7x + 6$.

Check your solution by substituting
$0 = 6^2 - 7(6) + 6$
$0 = 36 - 42 + 6$
$0 = 0$

$x^2 + 7x + 12 = 0$

$(x + \_\_)(x + \_\_) = 0$

Factoring

Name MissMcGinnis

All About Quadratics

All quadratic equations have a term containing x^2.

A quadratic FUNCTION in standard form is $ax^2+bx+c=0$.

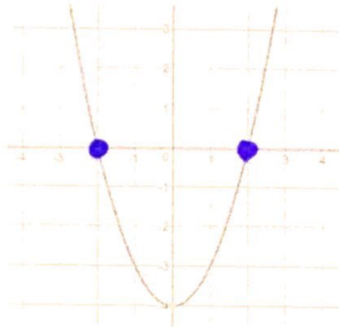
It forms a u-shaped curve called a parabola.

The ZEROS of a quadratic function are the x-coordinates of the x-intercepts of the graph of the function.

These x-values are also the SOLUTIONS or ROOTS of the related quadratic EQUATION $ax^2 + bx + c = 0$.

Identify the ZEROS of the function:

a) from a graph...



$x = -2$

$x = 2$

b) from a table...

x	$x^2 + 4x + 3$
−4	3
−3	0
−2	−1
−1	0
0	3

x-int

-y-int

$x = -1$

$x = -3$

c) from the factors... $y = (x + 1)(2x - 3)$

$x + 1 = 0$ $\quad$ $2x - 3 = 0$

$\boxed{x = -1}$ $\quad$ $\boxed{x = \frac{3}{2}}$ $\quad$ $\frac{2x}{2} = \frac{3}{2}$

d) from the equation... $y = x^2 + 7x + 12$

$0 = x^2 + 7x + 12$

12, 3, 4, 7

$0 = (x + 3)(x + 4)$

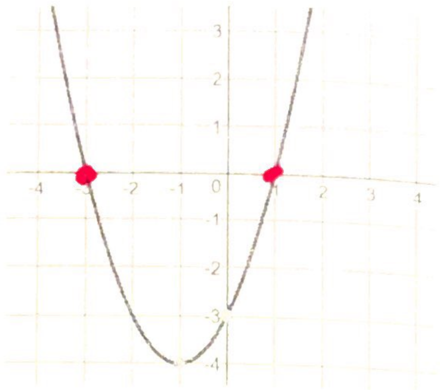
$x + 3 = 0$ $\quad$ $x + 4 = 0$

$\;\;-3 \;\; -3$ $\quad$ $\;\;-4\;\;-4$

$\boxed{x = -3}$ $\quad$ $\boxed{x = -4}$

Factors and Zeros

Given the graph



a) Identify the zeros of the function shown.

$x = -3 \qquad x = 1$

b) Write the function in factored form.

$y = (x+3)(x-1)$

c) Write the function in standard form.

$y = x^2 - x + 3x - 3 \qquad \boxed{y = x^2 + 2x - 3}$

The FACTORS of a function and it's ZEROS are OPPOSITES!

Challenge: Write an equation in standard form of a quadratic function with the zeros -4 and 2.

$y = (x+4)(x-2)$

$= x^2 - 2x + 4x - 8$

$\boxed{y = x^2 + 2x - 8}$

Solving Quadratic Equations by Factoring

Step 1: Make the equation = 0

Step 2: Look for a GCF! It goes outside...

Step 3: Factor with sum & product patterns, slide and divide if necessary.

Step 4: Set each factor = 0 and solve

a) $5x^2 - 15x = 0$

$\underbrace{5x}\underbrace{(x-3)} = 0$

$5x = 0 \qquad x - 3 = 0$

$\boxed{x = 0} \qquad \boxed{x = 3}$

b) $x^2 - 3x - 10 = 0$

-10

-5 2

-3

$\underbrace{(x-5)}\underbrace{(x+2)} = 0$

$x - 5 = 0 \qquad x + 2 = 0$

$\boxed{x = 5} \qquad \boxed{x = -2}$

c) $6x^2 + 7x = 3$

a·c

-18

$\frac{3}{2} = \frac{9}{6} \qquad \frac{-2}{6} = -\frac{1}{3}$

7

b

$6x^2 + 7x - 3 = 0$

$(2x+3)(3x-1) = 0$

$2x + 3 = 0 \qquad 3x - 1 = 0$

$2x = -3 \qquad 3x = 1$

$\boxed{x = -3/2} \qquad \boxed{x = 1/3}$

Solving by the Square Root Property

Use this method when the linear term is missing.

$$ax^2 \cancel{+ bx} + c = 0$$

Step 1: Isolate the x^2 or $(x+\#)^2$

Step 2: $\sqrt{\ }$ both sides, don't forget $\pm$!

Step 3: Solve for x

a) $5x^2 = 125$

$\frac{5x^2}{5} = \frac{125}{5}$

$\sqrt{x^2} = \sqrt{25}$

$\boxed{x = \pm 5}$

b) $2x^2 - 1 = 15$

$\frac{2x^2}{2} = \frac{16}{2}$

$\sqrt{x^2} = \sqrt{8}$

$\boxed{x = \pm 2\sqrt{2}}$

c) $\sqrt{(x+4)^2} = \sqrt{36}$

$x + 4 = \pm 6$

$x + 4 = 6$ $\quad$ $x + 4 = -6$

$\boxed{x = 2}$ $\quad$ $\boxed{x = -10}$

d) $5(x-1)^2 + 3 = 23$

$\frac{5(x-1)^2}{5} = \frac{20}{5}$

$\sqrt{(x-1)^2} = \sqrt{4}$

$x - 1 = \pm 2$

$x - 1 = 2$ $\quad$ $x - 1 = -2$

$\boxed{x = 3}$ $\quad$ $\boxed{x = -1}$

Solving by Completing the Square

Use this method when b is an even number.

Step 1: Get x^2 and x terms on the left and move the constants to the right; add a _ to both sides.

Step 2: Add a perfect square in both _'s

Step 3: Write the left side in $(\ )^2$ form and combine like terms on the right.

Half it, Drop it, Square it, Add it!

Step 4: $\sqrt{\ }$ both sides, don't forget $\pm$!

Step 5: Solve for x. Add and subtract to get 2 solutions.

a) $x^2 - 15 = 6x + 1$

$-6x + 15 \quad -6x \quad +15$

$x^2 - 6x + 9 = 16 + 9$

$\sqrt{(x-3)^2} = \sqrt{25}$

$x - 3 = \pm 5$

$x - 3 = 5$ $\quad$ $x - 3 = -5$

$\boxed{x = 8}$ $\quad$ $\boxed{x = -2}$

b) $x^2 + 8x + 10 = 0$

$x^2 + 8x + 16 = -10 + 16$

$\sqrt{(x+4)^2} = \sqrt{6}$

$x + 4 = \pm\sqrt{6}$

$\boxed{x = -4 \pm \sqrt{6}}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Can be used to solve ANY quadratic equation of the form $ax^2 + bx + c = 0$.

Step 1: Make the equation = 0

Step 2: Identify a, b, & c and substitute them into the formula.

Step 3: Simplify $b^2 - 4ac$ and $2a$.

Step 4: Simplify the $\sqrt{}$

Step 5: Add and subtract – then divide to find 2 solutions.

a) $2x^2 + 6x + 3 = 0$

$a = 2 \quad b = 6 \quad c = 3$

$$x = \frac{-6 \pm \sqrt{36 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{12}}{4}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{4}$$

$$\boxed{x = \frac{-3 \pm \sqrt{3}}{2}}$$

b) $2x^2 - x = 3$

$2x^2 - x - 3 = 0$

$a = 2 \quad b = -1 \quad c = -3$

$$x = \frac{1 \pm \sqrt{1 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{25}}{4}$$

$$x = \frac{1 \pm 5}{4}$$

$6/4 = \boxed{\frac{3}{2}}$

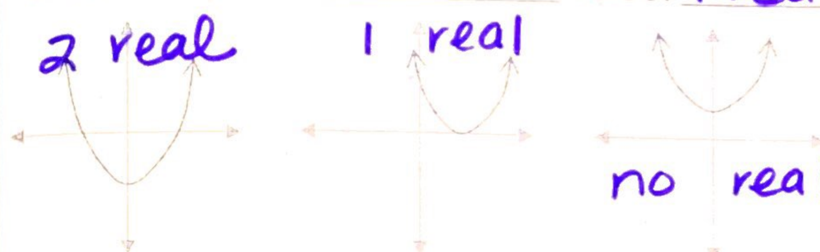
$\frac{-4}{4} = \boxed{-1}$

Discriminant

$$b^2 - 4ac$$

Allows us to determine the number of real roots for a quadratic equation in the form $ax^2 + bx + c = 0$

- if $b^2 - 4ac$ has a $+$ value: 2 real
- if $b^2 - 4ac$ has a $-$ value: no real
- if $b^2 - 4ac$ has a 0 value: 1 real



a) $x^2 + 4x + 16 = 0$

$a = 1 \quad b = 4 \quad c = 16$

$4^2 - 4(1)(16)$

$16 - 64 = -48$

discriminant = -48

# of real roots: None

b) $2x^2 + 8x + 8 = 0$

$a = 2 \quad b = 8 \quad c = 8$

$64 - 4(2)(8)$

$64 - 64 = 0$

discriminant = 0

# of real roots: 1

*Also, if the discriminant is a perfect square number, then the quadratic is factorable!

Can you determine which strategy to use for each?

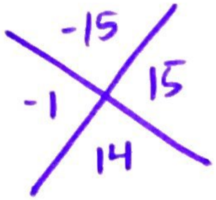
Choose **factoring, square root property, completing the square** or **quadratic formula**, and solve each equation.

1. $t^2 + 14t - 10 = 5$

$t^2 + 14t - 15 = 0$

$(t-1)(t+15) = 0$

$\boxed{t = 1}$ $\boxed{t = -15}$



2. $b^2 - 6b + 8 = 0$ [Factoring works too!] just make sure you got the same answers !!

$b^2 - 6b + \underline{9} = -8 + \underline{9}$

$\sqrt{(b-3)^2} = \sqrt{1}$

$b - 3 = \pm 1$

$b = 3 + 1$ $\boxed{b = 4}$ $b = 3 - 1$ $\boxed{b = 2}$

3. $3a^2 - 9a = 0$ GCF

$3a(a-3) = 0$

$3a = 0$ $\boxed{a = 3}$ $a - 3 = 0$ $\boxed{a = 3}$

4. $x^2 - 4x = 21$ Factoring works too!

$x - 4x + \underline{4} = 21 + \underline{4}$

$\sqrt{(x-2)^2} = \sqrt{25}$

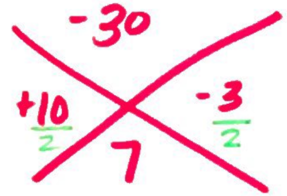
$x - 2 = \pm 5$

$x - 2 = 5$ $\boxed{x = 7}$ $x - 2 = -5$ $\boxed{x = -3}$

5. $2x^2 + 7x - 15 = 0$ Slide & Divide or Quad Formula

$(2x - 3)(x + 5) = 0$

$\boxed{x = \frac{3}{2}}$ $\boxed{x = -5}$



6. $p^2 + 4p + 1 = 0$ Quad Formula or CTS

$a = 1$ $b = 4$ $c = 1$

$x = \dfrac{-4 \pm \sqrt{16 - 4(1)(1)}}{2}$

$x = \dfrac{-4 \pm \sqrt{12}}{2} = \dfrac{-4 \pm 2\sqrt{3}}{2} =$ $\boxed{x = -2 + \sqrt{3}}$ $\boxed{x = -2 - \sqrt{3}}$

7. $2(x+3)^2 - 1 = 71$

$$\frac{2(x+3)^2}{2} = \frac{72}{2}$$

$$\sqrt{(x+3)^2} = \sqrt{36}$$

$$x+3 = \pm 6$$

$x+3=6$

$\boxed{x=3}$

$x+3=-6$

$\boxed{x=-9}$

8. Given the equation $x^2 - 2x - 15 = 0$

What are the factors of the equation?

$(x-5)(x+3)=0$

What are the roots of the equation?

$x=5 \quad x=-3$

What are the zeros of the equation?

$x=5 \quad x=-3$

9. Using the discriminant, determine which of the following quadratic equations has one real root. Check all that apply.

- ✓ $x^2+4x+4=0$ $\quad 4^2-4(1)(4)=0$ ✓
- ☐ $x^2+9x+10=0$
- ☐ $2x^2-15x+3=0$
- ✓ $3x^2-18x+27=0$ $\quad (-18)^2-4(3)(27)$
  $324 - 324 = 0$ ✓

10. Write a quadratic function in standard form with zeros of 4 and 0.

$x=4 \quad x=0$

$y = x(x-4)$

$\boxed{y = x^2-4x}$