

Unit 3

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Ratios in Similar Polygons

Fill in the blanks to complete each definition.

1. A similarity ratio is the ratio of the lengths of the corresponding sides of two similar polygons.
2. Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.
3. Figures that are similar have the same shape but not necessarily the same Size.

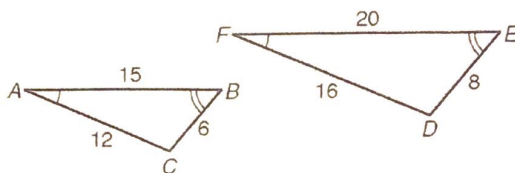
Use the figure for Exercises 4 and 5. The triangles are similar.

4. Name the pairs of congruent angles.

$$\angle A \cong \angle F$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle D$$



5. Write the corresponding side lengths in the proportion.

$$\frac{AB}{FE} = \frac{CB}{DE} = \frac{AC}{FD} = \frac{15}{20} = \frac{3}{4}$$

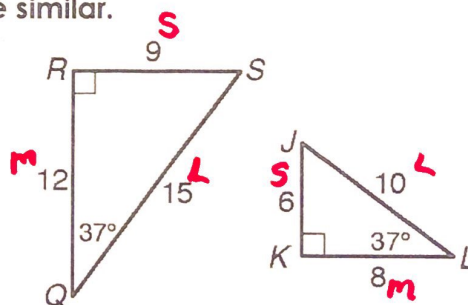
Use the figure to the right for Exercises 6 and 7. The triangles are similar.

6. Circle the correct similarity statement.

$\triangle QRS \sim \triangle KJL$ $\triangle RSQ \sim \triangle KJL$ $\triangle QSR \sim \triangle LKJ$

7. Write the corresponding side lengths in the proportion.

$$\frac{RS}{KJ} = \frac{RQ}{KL} = \frac{SQ}{JL}$$



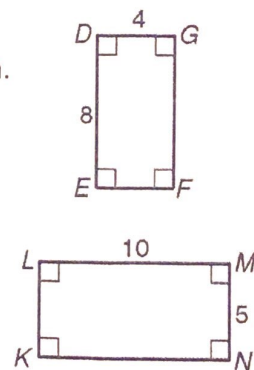
Use the figure to the right for Exercise 8.

8. Substitute numbers for the side lengths and reduce each ratio to simplest form.

$$\frac{DG}{MN} = \frac{4}{5} \checkmark$$

$K = \frac{\text{New}}{\text{original}}$

$$\frac{DE}{LM} = \frac{8}{10} = \frac{4}{5} \checkmark$$



Scale Factor

Scale Factor – the ratio of corresponding sides

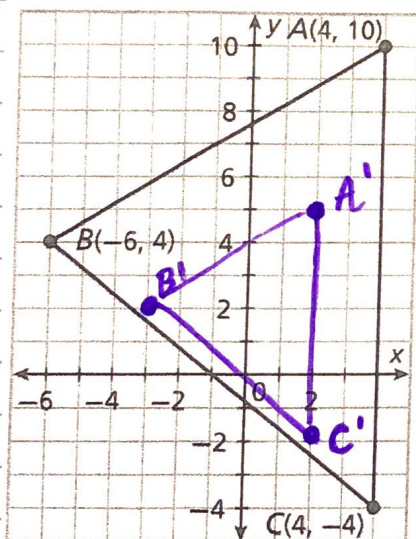
- When scale factor is greater than 1, the shape gets *bigger* and this is called an enlargement.
- When scale factor is less than 1, but greater than 0, the shape gets *smaller* and this is called a reduction.
- Formula: $\frac{\text{New}}{\text{Original}} = \frac{\text{Image}}{\text{Preimage}}$

Dilations

Apply the dilation D to the polygon with the given vertices. Name the coordinates of the image points. Identify and describe the transformation as an enlargement or reduction.

9. $D(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

$A(4, 10)$, $B(-6, 4)$, and $C(4, -4)$



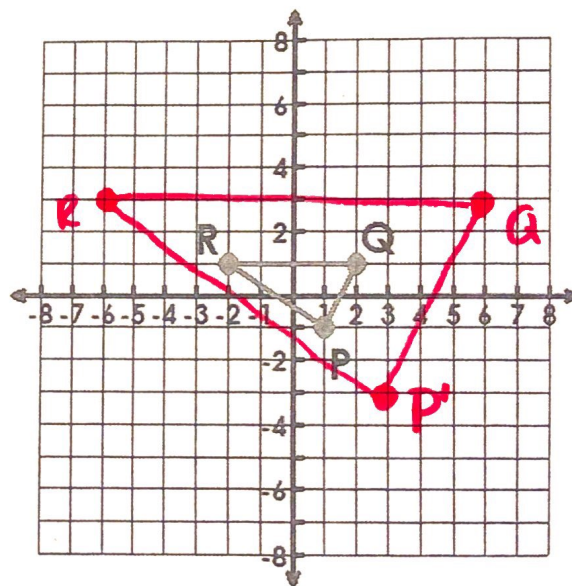
$A'(2, 5)$, $B'(-3, 2)$, and $C'(2, -2)$

This shape is a/n reduction.

The scale factor is $\frac{1}{2}$.

10. $D(x, y) \rightarrow (3x, 3y)$

$P(1, -1)$, $Q(2, 1)$, $R(-2, 1)$



$P'(3, -3)$, $Q'(6, 3)$, and $R'(-6, 3)$

This shape is a/n enlargement.

The scale factor is 3.

Proportionality Theorem

Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

Also known as the "side splitter theorem"

Example:

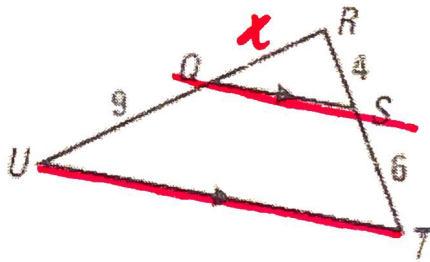
Determine the length of RQ:

Example 1:

$$\frac{x}{9} = \frac{4}{6}$$

$$6x = 36$$

$$\boxed{x = 6}$$



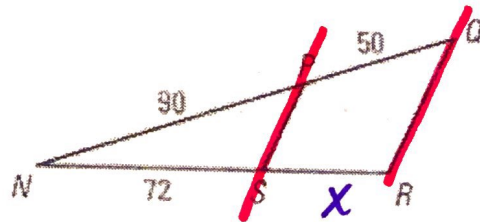
$PS \parallel QR$ Determine the Length of SR:

Example 2:

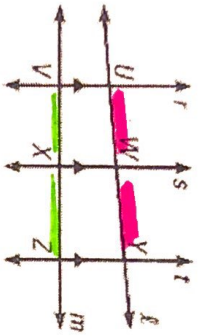
$$\frac{90}{50} = \frac{72}{x}$$

$$90x = 3600$$

$$\boxed{x = 40}$$



The same is true for transversals cut by parallel lines.

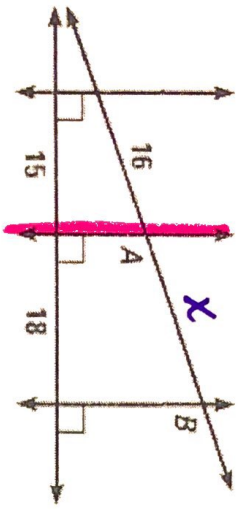


$$\frac{UW}{WV} = \frac{VX}{XZ}$$

Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Example: find the length of AB.



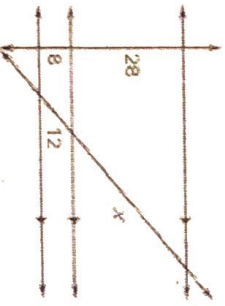
$$\frac{x}{16} = \frac{18}{15}$$

$$288 = 15x$$

$$\boxed{19.2 = x}$$

Note Practice:

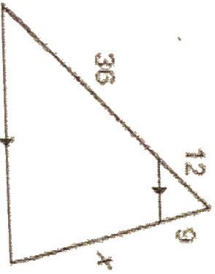
Solve for x.



$$\frac{x}{12} = \frac{28}{8}$$

$$8x = 336$$

$$\boxed{x = 42}$$



$$\frac{x}{9} = \frac{36}{12}$$

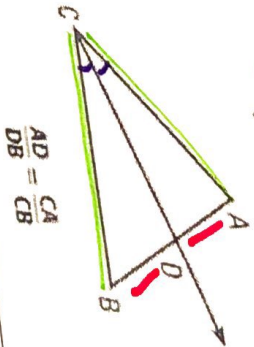
$$12x = 324$$

$$\boxed{x = 27}$$

Angle Bisector Theorem

Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$$\frac{AD}{DB} = \frac{CA}{CB}$$

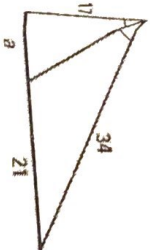
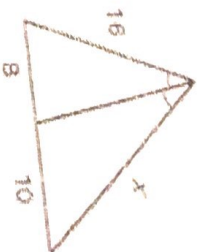
Examples: solve for the missing variable

$$\frac{10}{8} = \frac{x}{16}$$

$$160 = 8x$$

$$\boxed{20 = x}$$

Find AB:



$$\frac{a}{21} = \frac{17}{34}$$

$$34a = 357$$

$$\boxed{a = 10.5}$$

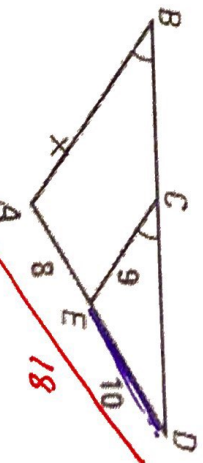
Find AB:

* solve for x. AB is parallel to CE

$$\frac{52}{20} = \frac{x}{30}$$

$$20x = 1560$$

$$\boxed{x = 78}$$



$$\frac{10}{18} = \frac{9}{x}$$

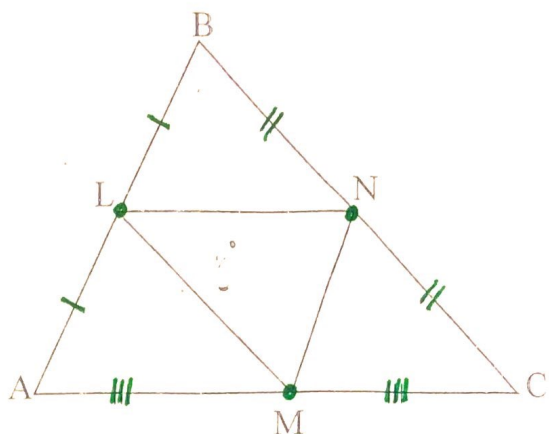
$$10x = 162$$

$$\boxed{x = 16.2}$$

Triangle Midsegment and Proportionality Theorem

Triangle Midsegment Theorem: The segment connecting the midpoints of two sides of the triangle is parallel to the third side and half the length of the third side.

Use $\triangle ABC$, where L, M, and N are midpoints of the sides.



1. $\overline{LM} \parallel \overline{BC}$

2. $\overline{AB} \parallel \overline{NM}$

3. If $AC = 20$, then $LN = \underline{10}$

4. If $MN = 7$, then $AB = \underline{14}$

5. If $NC = 9$, then $LM = \underline{9}$

6. If $LM = 3x + 7$, and $BC = 7x + 6$, then $LM = \underline{31}$

$$2(3x + 7) = 7x + 6$$

$$6x + 14 = 7x + 6$$

$$\boxed{8 = x}$$

$$3(8) + 7 = 31$$

7. If $MN = x - 1$, and $AB = 6x - 18$, then $AB = \underline{6}$

$$2(x - 1) = 6x - 18$$

$$2x - 2 = 6x - 18$$

$$16 = 4x$$

$$\boxed{x = 4}$$

$$6(4) - 18$$

$$24 - 18$$

$$6$$

8. Find each measure. H, G, and I are all midpoints.

a) $HI = \underline{9.1}$

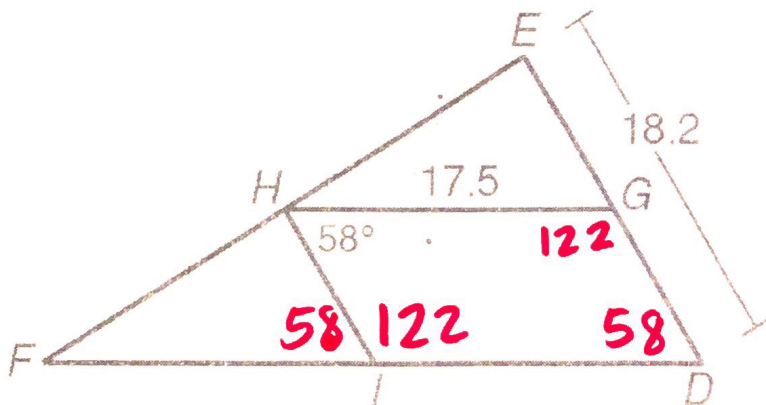
b) $DF = \underline{35}$

c) $GE = \underline{9.1}$

d) $m\angle HIF = \underline{58^\circ}$

e) $m\angle HGD = \underline{122}$

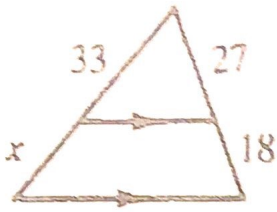
f) $m\angle D = \underline{58^\circ}$



Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

Find the value of x :

9.

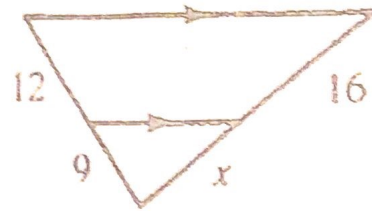


$$\frac{33}{x} = \frac{27}{18}$$

$$\frac{27x}{27} = \frac{594}{27}$$

$$\boxed{x = 22}$$

10.

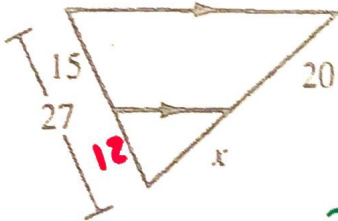


$$\frac{12}{9} = \frac{16}{x}$$

$$144 = 12x$$

$$\boxed{x = 12}$$

11.



$$240 = 15x$$

$$\frac{15}{12} = \frac{20}{x}$$

$$\boxed{x = 16}$$

12.

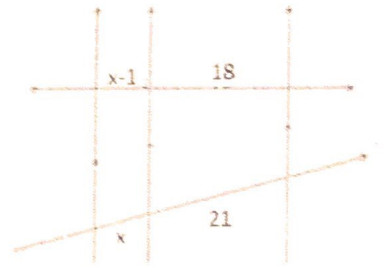
$$\frac{18}{x-1} = \frac{21}{x}$$

$$21(x-1) = 18x$$

$$21x - 21 = 18x$$

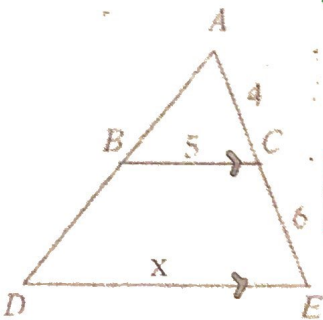
$$3x = 21$$

$$\boxed{x = 7}$$



*

13.



$$\frac{5}{x} = \frac{4}{10}$$

$$50 = 4x$$

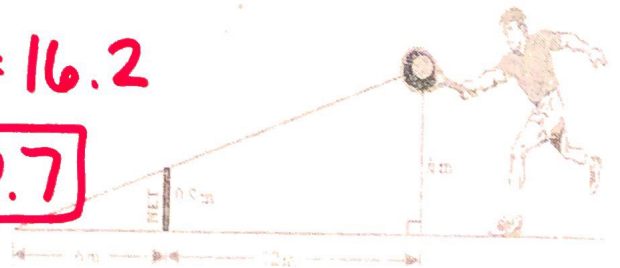
$$\boxed{12.5 = x}$$

*

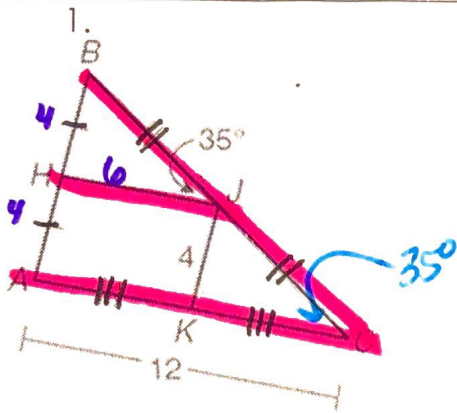
$$14. \frac{6}{18} = \frac{9}{x}$$

$$6x = 16.2$$

$$\boxed{x = 2.7}$$



Midsegment vs. Proportionality Thm.

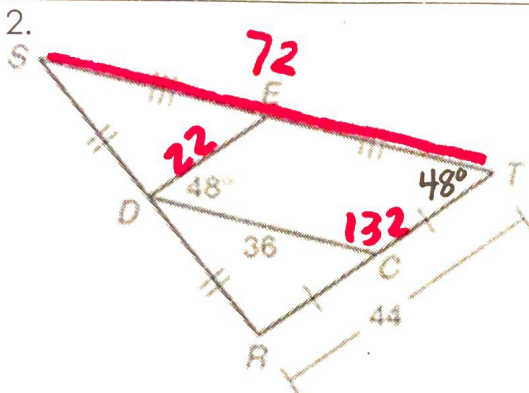


1. Problem Type: midsegment

a. Find $AB = 8$

b. Find measure of angle ACB

$m\angle ACB = 35^\circ$



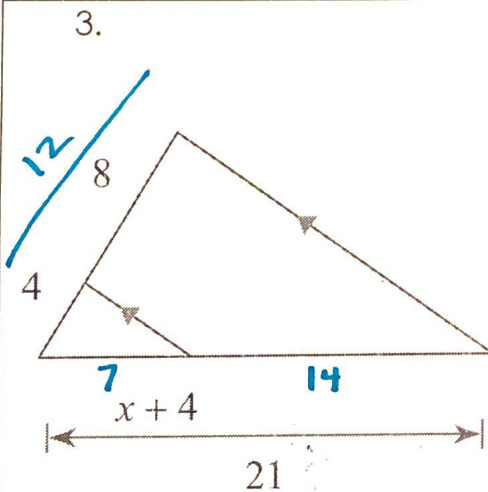
2. Type of Problem: midsegment

a. Measure of angle STR: 48°

b. Length of DE: 22

c. Length of ST: 72

d. Measure of angle DCT: 132°



3. Type of Problem: Proportionality Thm.

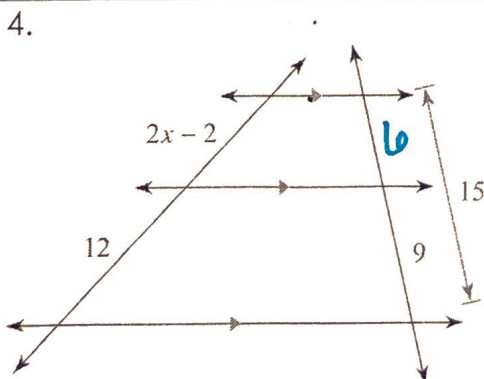
Solve for x:

$$\frac{12}{4} = \frac{21}{x+4}$$

$$12x + 48 = 84$$

$$12x = 36$$

$$x = 3$$



4. Type of Problem: Proportionality Thm

Solve for x:

$$\frac{2x-2}{12} = \frac{6}{9}$$

$$18x - 18 = 72$$

$$18x = 90$$

$$x = 5$$

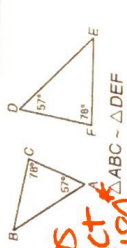
Similarity

Ways to Prove Triangles are Similar

AA- Postulate:

If two angles of one triangle are congruent to two angles of another, then the triangles are similar.

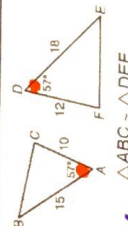
Parallel lines ***vertical*** ***Always Subtract from 180***



SAS- Postulate:

If one angle of one triangle is congruent to the one angle of another triangle and the adjacent sides are proportional, then the triangles are similar.

$$\frac{15}{18} = \frac{10}{12} = \frac{5}{6} \checkmark \quad \frac{5}{6} \checkmark \quad .83 \checkmark$$



Show Scale Factor

SSS- Postulate:

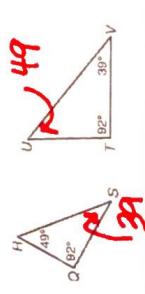
If all three sides of one triangle are proportional to corresponding sides of another triangle, then the triangles are similar.

$$\frac{5}{15} = \frac{6}{18} = \frac{12}{24} = \frac{1}{3} \checkmark \quad \frac{12}{14.4} = \frac{10}{14.4} = \frac{5}{7.2} \checkmark \quad \frac{12}{14.4} = \frac{10}{14.4} \checkmark$$

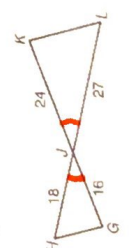


Practice: Explain why the triangles are similar (SSS~, SAS~, or AA~) and write a similarity statement.

1) $\triangle RQS \sim \triangle TV$ by **AA**

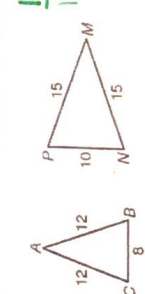


2) $\triangle HGJ \sim \triangle LKJ$ by **SAS**



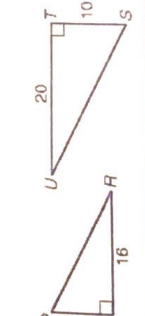
$$\frac{18}{27} = \frac{16}{24} = \frac{2}{3} \checkmark \quad 432 = 432$$

3) $\triangle ABC \sim \triangle MNP$ by **SSS**



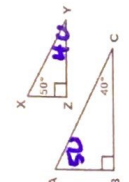
$$\frac{12}{10} = \frac{12}{15} = \frac{8}{10} = \frac{4}{5} \checkmark$$

5) $\triangle QPR \sim \triangle TSU$ by **SAS**



$$\frac{16}{20} = \frac{8}{10} = \frac{4}{5} \checkmark$$

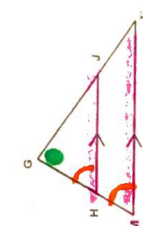
6) $\triangle ABC \sim \triangle XZY$ by **AA**



$$\frac{2}{4} = \frac{3}{6} = \frac{1}{2} \checkmark$$

Postulates

7) $\triangle GHJ \sim \triangle MK$ by **AA**



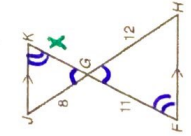
8) $\triangle AEF \sim \triangle ABC$ by **AA**



Reflexive!

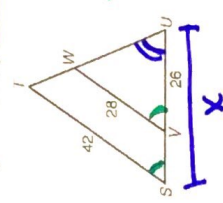
Explain why the triangles are similar (SSS~, SAS~, or AA~) and find each length.

9) Similar by **AA** and **GK = 7.3**



$$\frac{8}{12} = \frac{x}{11} \quad 12x = 88 \quad x = 7.3$$

10) Similar by **AA** and **SU = 39**



$$\frac{42}{28} = \frac{x}{26} \quad 28x = 1092 \quad x = 39$$

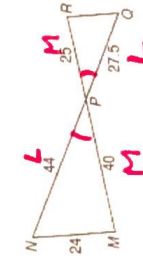
11) Similar by **AA** and **DE = 9.75**



$$13 - 3.25 = x \quad 9.75 = x$$

$$\frac{5}{1.25} = \frac{y}{3.25} \quad 1.25y = 16.25 \quad y = 13$$

12) Similar by **SAS** and **RQ = 15**

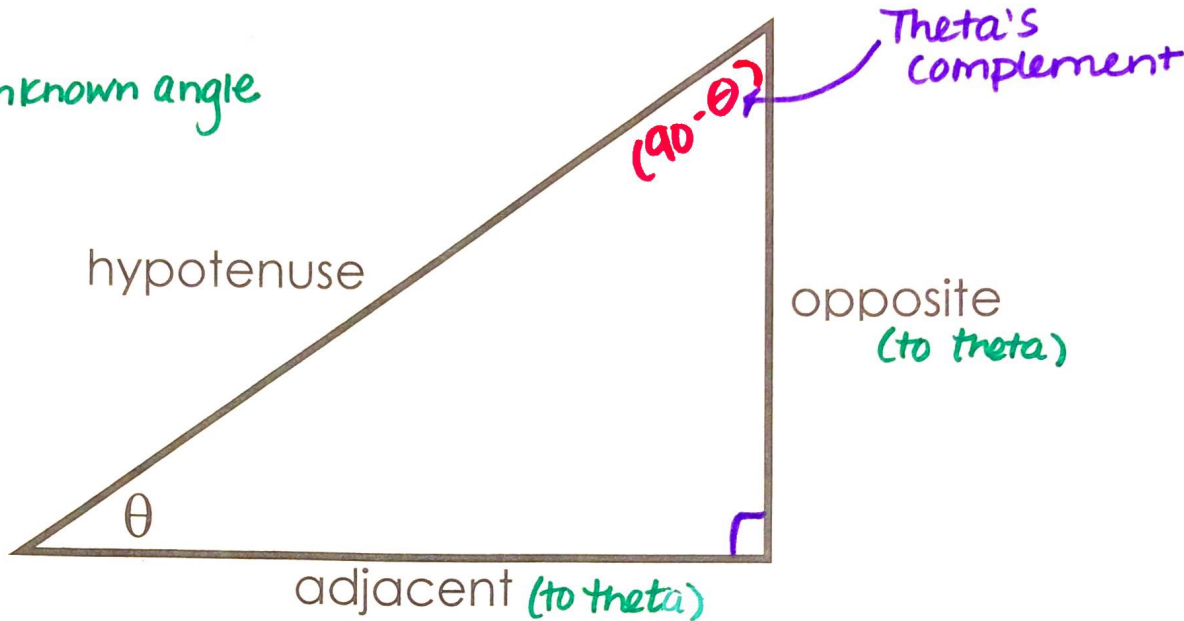


$$\frac{40}{25} = \frac{44}{27.5} \quad X = 15$$

$$\frac{40}{25} = \frac{24}{X} \quad 1.6 = 1.6 \checkmark$$

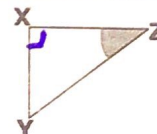
Trigonometry Ratios

$\theta \rightarrow$ theta
represents unknown angle



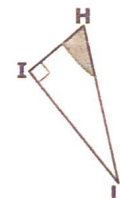
Sine	Cosine	Tangent
<p>$\sin(\theta) = \frac{3}{5}$</p>	<p>$\cos(\theta) = \frac{4}{5}$</p>	<p>$\tan(\theta) = \frac{3}{4}$</p>
SOH	CAH	TOA
$S = \frac{O}{H}$	$C = \frac{A}{H}$	$T = \frac{O}{A}$

1. Identify the side that is opposite $\angle Z$ \overline{XY}



2. Identify the side that is adjacent to $\angle Z$ \overline{XZ}

3. Identify the side that is opposite $\angle H$ \overline{IV}

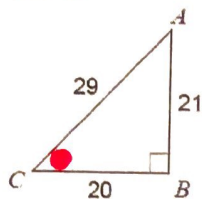


4. Identify the side that is adjacent to $\angle H$ \overline{IH}

SOH CAH TOA

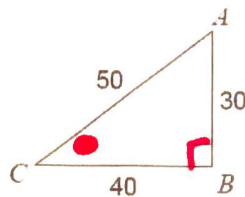
Find the value of each trigonometric ratio. Express your answer as a fraction in lowest terms.

5. $\sin C =$



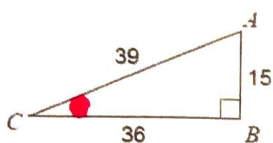
$$\frac{21}{29}$$

6. $\sin C =$



$$\frac{30}{50} = \frac{3}{5}$$

7. $\cos C =$



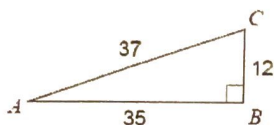
$$\frac{36}{39} = \frac{12}{13}$$

8. $\cos C =$



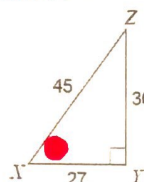
$$\frac{8}{17}$$

9. $\tan A =$



$$\frac{12}{35}$$

10. $\tan X =$



$$\frac{36}{27} = \frac{4}{3}$$

For each of the following find the trigonometric ratio.

11. $\sin A$

$$\frac{15}{17}$$

12. $\cos A$

$$\frac{8}{17}$$

13. $\tan A$

$$\frac{15}{8}$$

14. $\sin B$

$$\frac{8}{17}$$

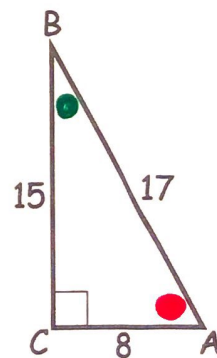
15. $\cos B$

$$\frac{15}{17}$$

16. $\tan B$

$$\frac{8}{15}$$

The tangents of complementary angles will be reciprocals

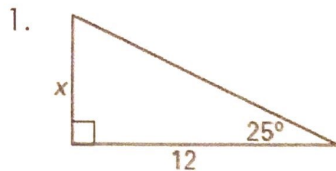


How do your answers in #11-13 compare to those in #14-16?

* The sine of an angle is ALWAYS equal to the cosine of its complement *

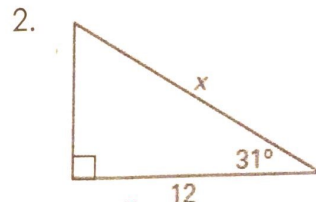
Missing Sides & Angles

Using Trig Ratios to find Missing Sides or Angles

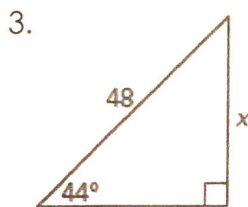


$$\tan(25) = \frac{x}{12}$$
$$12 \cdot \tan(25) = x$$

$$\boxed{x = 5.6}$$



$$\cos(31) = \frac{12}{x} \quad x = \frac{12}{\cos(31)}$$
$$\boxed{x = 14}$$



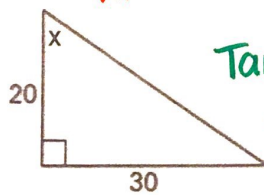
$$\sin(44) = \frac{x}{48}$$
$$48 \cdot \sin(44) = x$$
$$\boxed{x = 33.3}$$



$$\tan(67) = \frac{18}{x}$$
$$x = \frac{18}{\tan(67)}$$

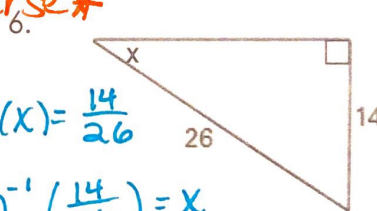
$$\boxed{x = 7.6}$$

5. * To find the angle use



$$\tan(x) = \frac{30}{20}$$
$$\tan^{-1}\left(\frac{30}{20}\right) = x$$
$$\boxed{56.3^\circ = x}$$

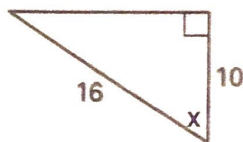
inverse*



$$\sin(x) = \frac{14}{26}$$
$$\sin^{-1}\left(\frac{14}{26}\right) = x$$

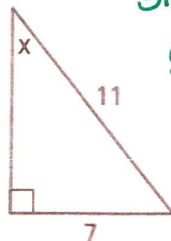
$$\boxed{32.6^\circ = x}$$

7.



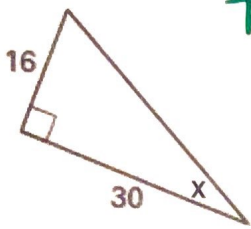
$$\cos(x) = \frac{10}{16}$$
$$\cos^{-1}\left(\frac{10}{16}\right) = x$$
$$\boxed{x = 51.3^\circ}$$

8.



$$\sin(x) = \frac{7}{11}$$
$$\sin^{-1}\left(\frac{7}{11}\right) = x$$
$$\boxed{39.5^\circ = x}$$

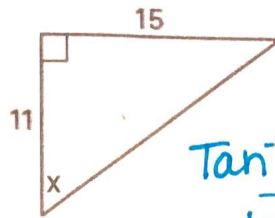
9.



$$\tan^{-1}\left(\frac{16}{30}\right) = x$$

$$\boxed{28.1^\circ = x}$$

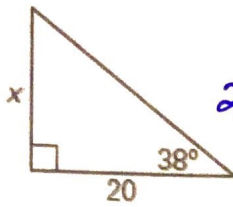
10.



$$\tan^{-1}\left(\frac{15}{11}\right) = x$$

$$\boxed{53.7^\circ = x}$$

11.

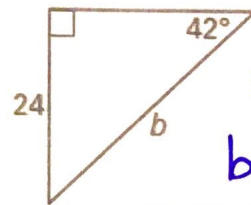


$$\tan(38) = \frac{x}{20}$$

$$20 \cdot \tan(38) = x$$

$$\boxed{15.6 = x}$$

12.

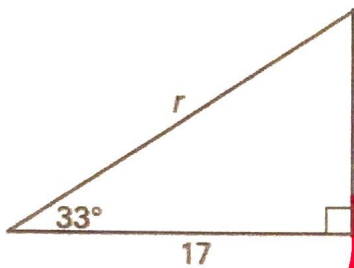


$$\sin(42) = \frac{24}{b}$$

$$b = 24 / \sin(42)$$

$$\boxed{b = 35.9}$$

13.

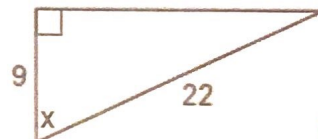


$$\cos(33) = \frac{17}{r}$$

$$r = \frac{17}{\cos(33)}$$

$$\boxed{r = 20.3}$$

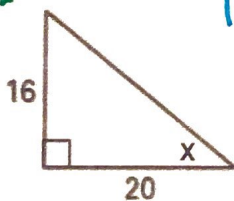
* 14.



$$\cos^{-1}\left(\frac{9}{22}\right) = x$$

$$\boxed{x = 65.9^\circ}$$

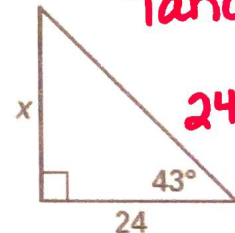
15. *



$$\tan^{-1}\left(\frac{16}{20}\right) = x$$

$$\boxed{38.7^\circ = x}$$

16.

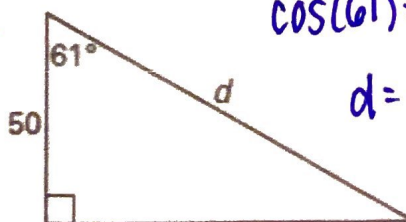


$$\tan(43) = \frac{x}{24}$$

$$24 \cdot \tan(43) = x$$

$$\boxed{22.4 = x}$$

17.

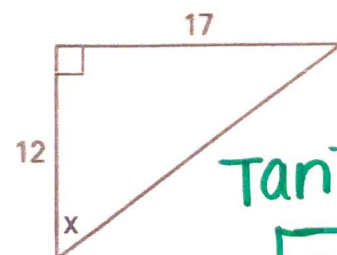


$$\cos(61) = \frac{50}{d}$$

$$d = \frac{50}{\cos(61)}$$

$$\boxed{d = 103.1}$$

* 18.

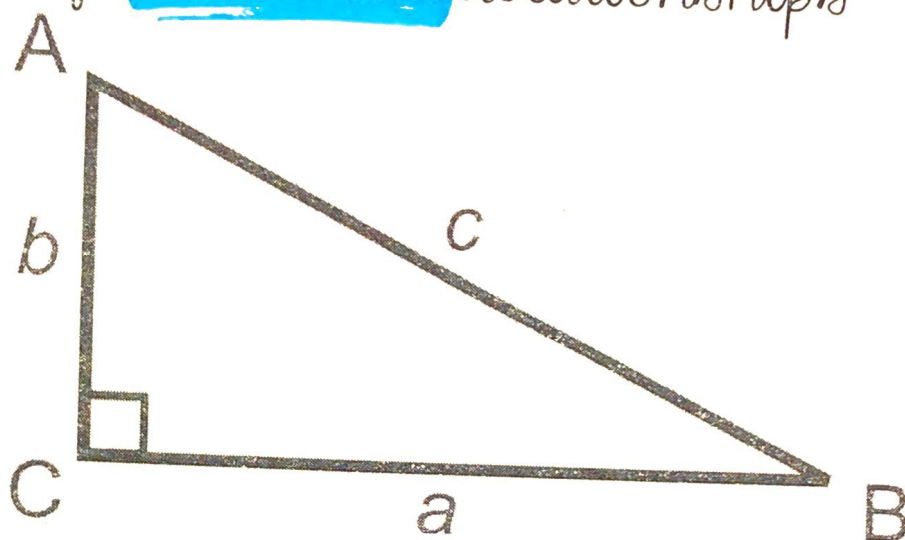


$$\tan^{-1}\left(\frac{17}{12}\right) = x$$

$$\boxed{54.8^\circ = x}$$



Trig CoFunction Relationships



In the above triangle, Name two complementary angles.

$\angle A$ & $\angle B$ are complementary

The Sine of an angle is always equal to the **cosine** of its complement.

So... for this Δ

$$\sin(A) = \cos(B)$$

And

$$\cos(A) = \sin(B)$$

The Tangents of complementary angles are **reciprocals** of each other.

So.. if the $\tan(A) = \frac{3}{5}$ then the $\tan(B) = \frac{5}{3}$

Example: If you are given an Angle:

$$\sin(\theta) = \cos(90 - \theta)$$

$90 - 75$

$$\sin(75) = \cos(15)$$

Example: If you are given a ratio.

Given: A and B are complementary. And

$$\sin(B) = \frac{4}{5}$$

Find:

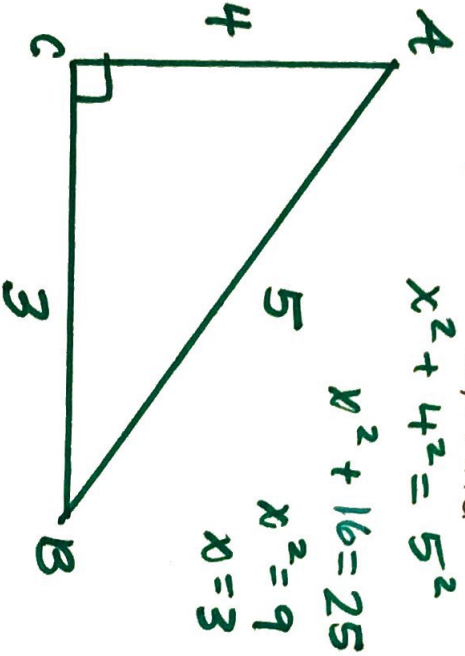
$$\sin(A) = \frac{3}{5}$$

$$\cos(A) = \frac{4}{5}$$

$$\cos(B) = \frac{3}{5}$$

$$\tan(A) = \frac{3}{4}$$

$$\tan(B) = \frac{4}{3}$$



1. $\cos(22) = \sin(68)$

$90 - 22$

2. Given: A and B are complementary. And

$$\cos(B) = \frac{8}{17}$$

Find:

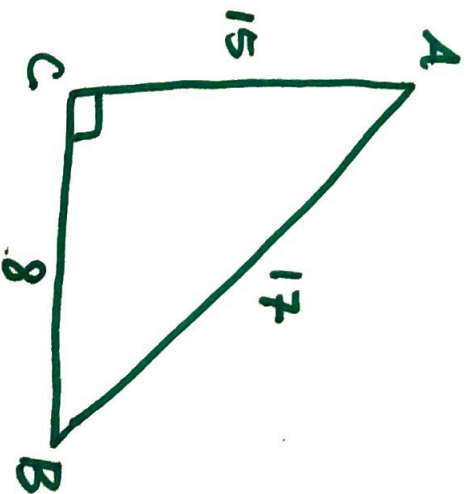
$$\sin(A) = \frac{8}{17}$$

$$\cos(A) = \frac{15}{17}$$

$$\sin(B) = \frac{15}{17}$$

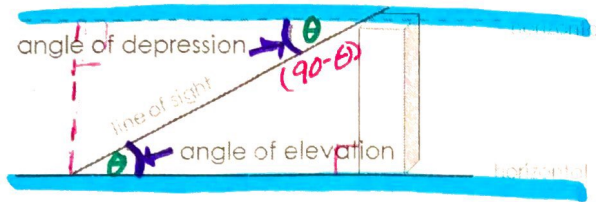
$$\tan(B) = \frac{15}{8}$$

$$\tan(A) = \frac{8}{15}$$

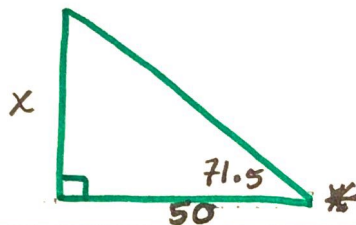


Triangle Application Problems – SOHCAHTOA

Angle of Elevation & Angle of Depression



1. A rock is 50 feet from the base of a large tree. The surveyor measures the angle of elevation from the rock to a bird on top of the tree as 71.5° . How tall is the tree?

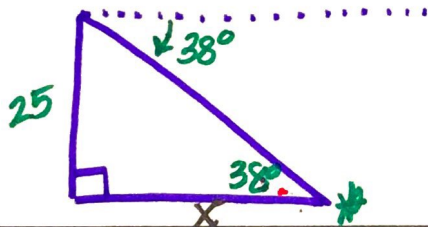


$$\tan(71.5) = \frac{x}{50}$$

$$50 \cdot \tan(71.5) = x$$

$$\boxed{149.4 = x}$$

2. The angle of depression from the top of a tower to a boulder on the ground is 38° . If the tower is 25m high, how far from the base of the tower is the boulder?

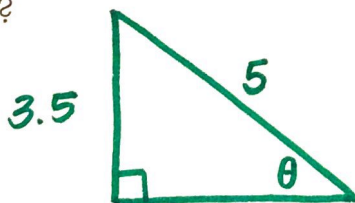


$$\tan(38) = \frac{25}{x}$$

$$x = \frac{25}{\tan(38)}$$

$$\boxed{x = 32}$$

3. A rocket is launched at an angle into outer space. After a minute, the rocket traveled 5 miles and had an altitude of 3.5 miles. What is the angle of elevation that the rocket was launched at?

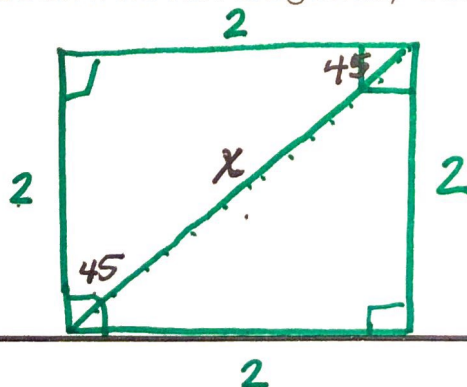


$$\sin^{-1}\left(\frac{3.5}{5}\right) = \theta$$

$$\boxed{44.4^\circ = \theta}$$

4. Tom went to a park that is the shape of a square. If he runs a total of 8 miles around the park, how far would it have been if he ran diagonally across the park?

$$\sin(45) = \frac{2}{x}$$



$$\cos(45) = \frac{2}{x}$$

$$x = \frac{2}{\cos(45)}$$

$$\boxed{x = 2.8}$$

Trig Quiz Review

1. Length AC 10.5

2. Length AB 18.3

3. $m\angle A$ 55° $90-35$

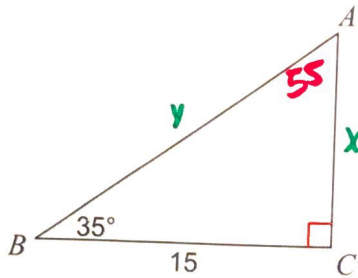
$$\tan(35) = \frac{x}{15}$$

$$15 \tan(35) = x$$

$$\cos(35) = \frac{15}{y}$$

$$y = \frac{15}{\cos(35)}$$

$$y = 18.3$$



4. Length of BC 12.3

5. $m\angle B$ 18.4°

$$\cos^{-1}\left(\frac{4.1}{13}\right) = A$$

6. $m\angle A$ 71.6°

$$71.6 = A$$

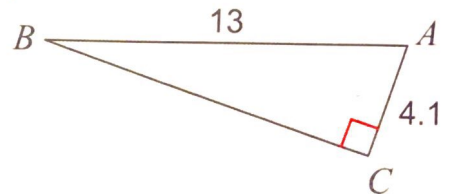
$$4.1^2 + a^2 = 13^2$$

$$16.81 + a^2 = 169$$

$$a^2 = 152.19$$

$$a = 12.3$$

$$\sin^{-1}\left(\frac{4.1}{13}\right) = B$$



7. $\sin(\theta) = \frac{12}{13}$

8. $\cos(90-\theta) = \frac{12}{13}$



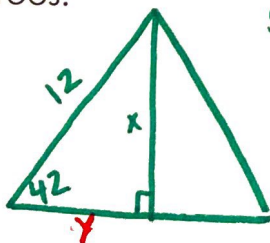
9. $\sin(52) = \cos(\underline{38})$ $90-52$

10. $\cos(32) = \sin(\underline{58})$ $90-32$

11. If $\tan(\theta) = \frac{8}{15}$ then

$$\tan(90-\theta) = \frac{15}{8}$$

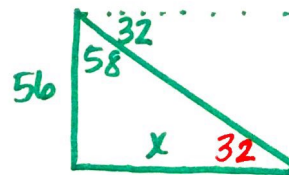
12. Find the height of an isosceles triangle if the legs of the triangle are 12in, and the base angles are 42 degrees.



$$\sin(42) = \frac{x}{12}$$

$$x = 8$$

13. The angle of depression from the top of a lighthouse to a ship at sea is 32 degrees. If the lighthouse is 56 feet tall, how far away from the base of the lighthouse is the ship?



$$\tan(58) = \frac{x}{56}$$

$$56 \tan(58) = x$$

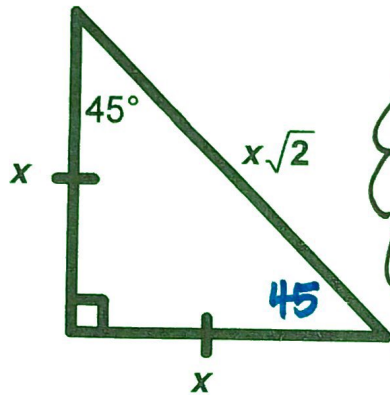
$$\boxed{89.6 \text{ ft}}$$

$$\tan(32) = \frac{56}{x}$$

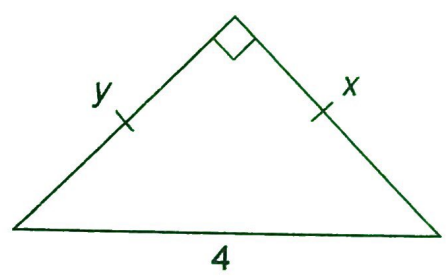
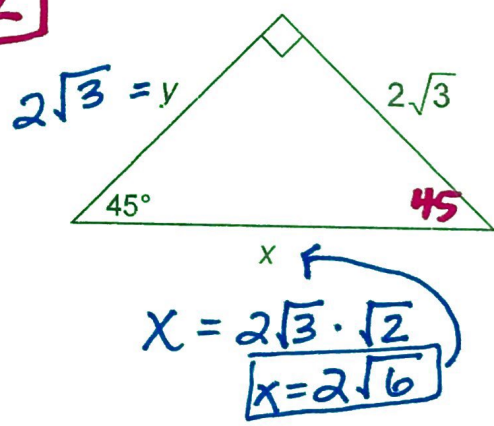
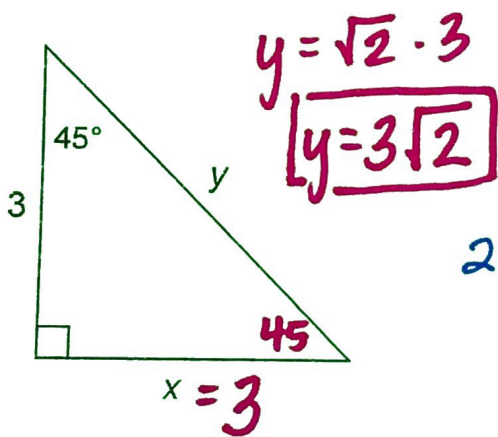
$$x = \frac{56}{\tan(32)}$$

SPECIAL

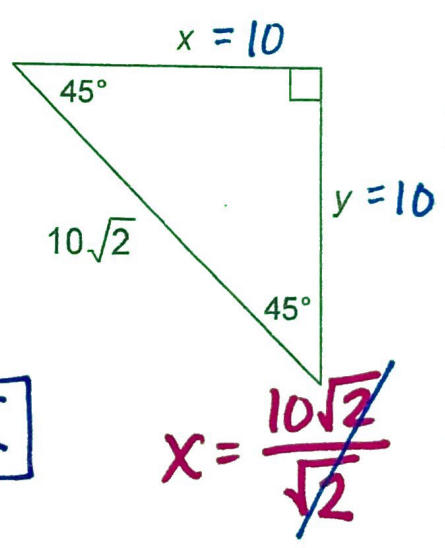
* Isosceles Right Triangle * 45°-45°-90° TRIANGLES



leg = leg
hyp = $\sqrt{2} \cdot \text{leg}$
leg = $\frac{\text{hyp}}{\sqrt{2}}$

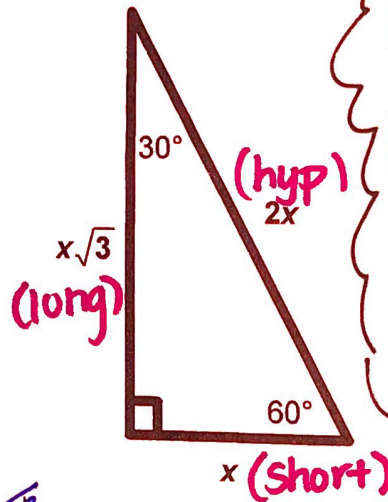


$y = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
 $y = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$
 $x = 2\sqrt{2}$

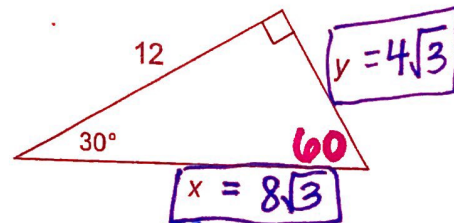
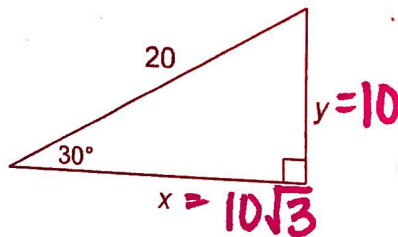
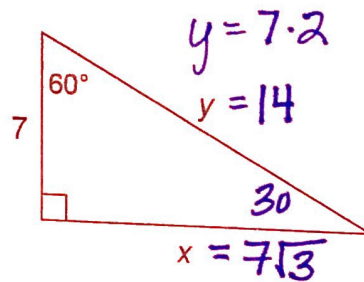
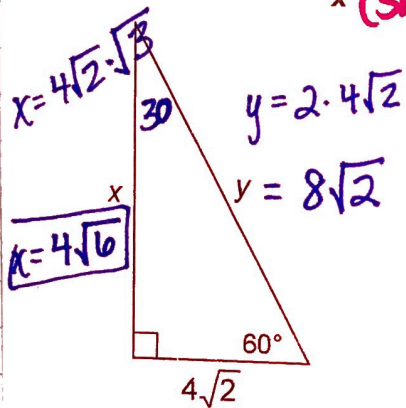


* Cases → SPECIAL RIGHT *

30°-60°-90° Triangles



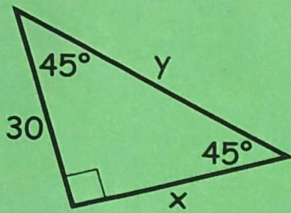
$hyp = short \cdot 2$
 $long = \sqrt{3} \cdot short$
 $short = \frac{hyp}{2}$
 $short = \frac{long}{\sqrt{3}}$



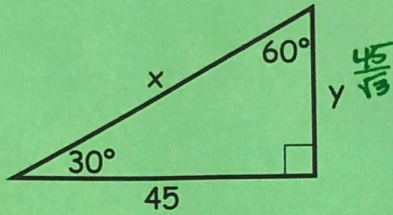
$y = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
 $y = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

Special Right

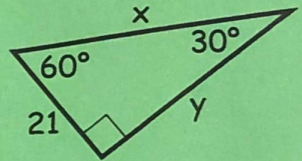
1. $x = 30, y = 30\sqrt{2}$



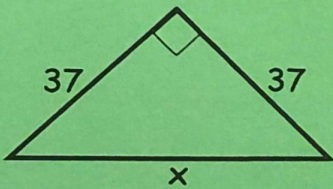
2. $x = 30\sqrt{3}, y = 15\sqrt{3}$



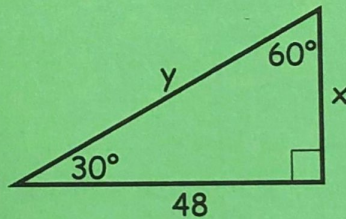
3. $x = 42, y = 21\sqrt{3}$



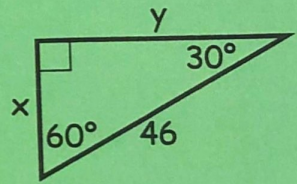
4. $x = 37\sqrt{2}$



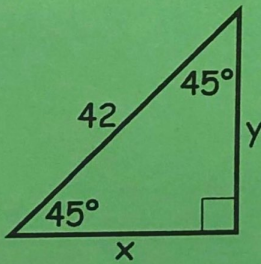
5. $x = 16\sqrt{3}, y = 32\sqrt{3}$



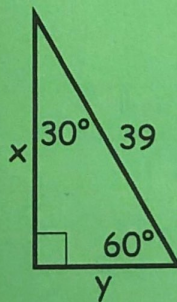
6. $x = 43, y = 43\sqrt{3}$



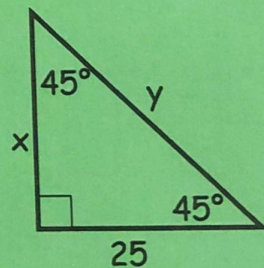
7. $x = 21\sqrt{2}, y = 21\sqrt{2}$



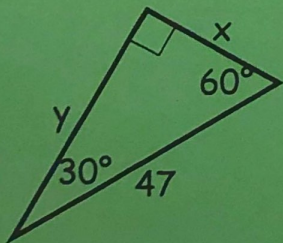
8. $x = \frac{39\sqrt{3}}{2}, y = \frac{39}{2}$



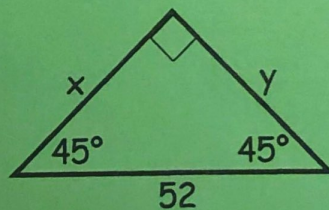
9. $x = 25, y = 25\sqrt{2}$



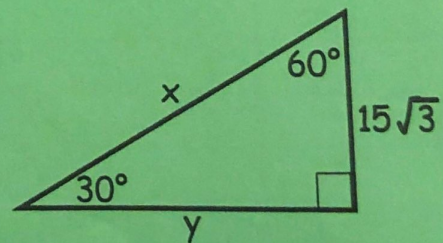
10. $x = \frac{47}{2}, y = \frac{47\sqrt{3}}{2}$



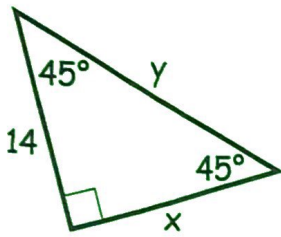
11. $x = 26\sqrt{2}, y = 26\sqrt{2}$



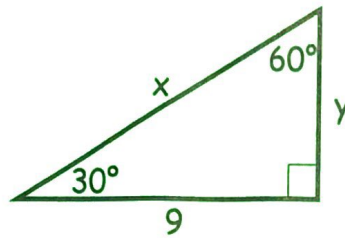
12. $x = 30\sqrt{3}, y = 45$



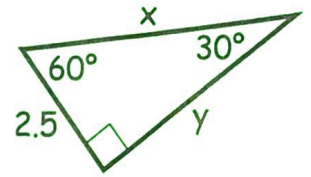
13. $x = 14$, $y = 14\sqrt{2}$



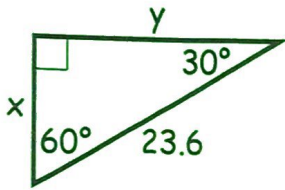
14. $x = 6\sqrt{3}$, $y = 3\sqrt{3}$



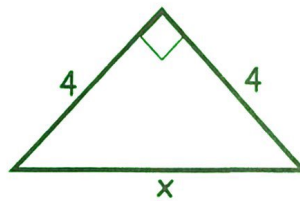
15. $x = 5$, $y = \frac{5\sqrt{3}}{2}$



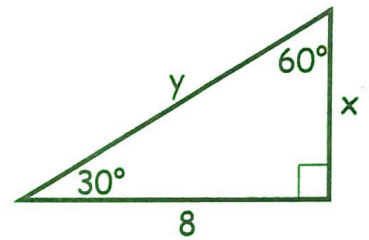
16. $x = 11.8$, $y = 11.8\sqrt{3}$



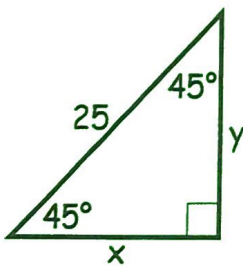
17. $x = 4\sqrt{2}$



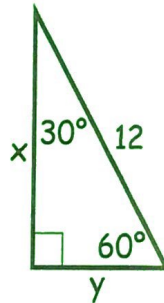
18. $x = \frac{8\sqrt{3}}{3}$, $y = \frac{16\sqrt{3}}{3}$



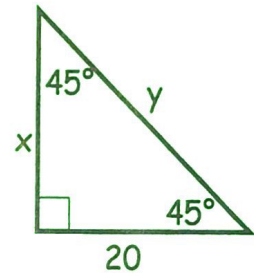
19. $x = \frac{25\sqrt{2}}{2}$, $y = \frac{25\sqrt{2}}{2}$



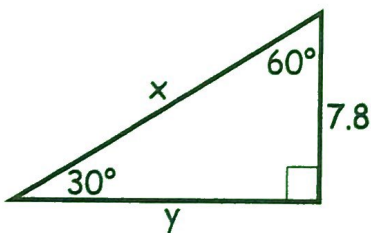
20. $x = 6\sqrt{3}$, $y = 6$



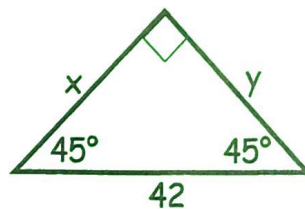
21. $x = 20$, $y = 20\sqrt{2}$



22. $x = 15.6$, $y = 7.8\sqrt{3}$



23. $x = 21\sqrt{2}$, $y = 21\sqrt{2}$



24. $x = 15$, $y = \sqrt{3}$

