

Unit 3A Factor & Solve

- ① GCF
- ② GCF daily ✓
- ③ Factor trinomials when $a = 1$ notes
- ④ $a = 1$ daily ✓
- ⑤ Factor trinomials $a > 1$ notes
- ⑥ $a > 1$ daily check
- ⑦ Special Cases Notes
- ⑧ ↓
- ⑨ Solve by Factoring Notes
- ⑩ Solve by Factoring w/
- ⑪ Solve by Square Roots
- ⑫ Solve by Complete the Square
- ⑬ Quad Formula daily check.

GCF

GCF Factoring

Greatest Common Factor: The greatest number that divides evenly into a set of numbers.

When dealing with variables, it is the lowest degree of a variable common to every term.

Examples: Factor the GCF of each of the following:

1. $(8xy - 2y)$ $2y(4x - 1)$	2. $(27x^3 - 9x^2)$ $9x^2(3x - 1)$	3. $(42xy^5 + 7x^2)$ $7x(6y^5 + x)$	4. $(2x^2 - 12x)$ $2x(x - 6)$
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To factor the GCF out of an expression, divide each term by the GCF and write your answer in un-distributed form.

Factor the GCF out of each of the following:

5. $10x^3 - 5x$ $5x(2x^2 - 1)$	6. $y^5 + y^2$ $y^2(y^3 + 1)$	7. $27x - 81xy^2$ $27x(1 - 3y^2)$	8. $10x - 14y + 40x$ $50x - 14y$ $2(25x - 7y)$
9. $x^3y - x^5yz^3 + x^2y^2$ $x^2y(x - x^3z^3 + y)$	10. $17z^2 - 68zy^2$ $17z(z - 4y^2)$	11. $2x - 16y$ $2(x - 8y)$	12. $5y + 20y^2 - 125$ $5(4y^2 + y - 25)$

GCF

daily ✓

GCF Factoring Daily Check

1. $\frac{9n^3}{3n} + \frac{15n^2}{3n} + \frac{30n}{3n}$

$$3n(3n^2 + 5n + 10)$$

2. $10x^3 - 6x^2 - 6x$

$$2x(5x^2 - 3x - 3)$$

3. $-50x^5y - 5xy$

$$-5xy(10x^4 + 1)$$

4. $-6a^4b + 24a^3b + 60a^3$

$$-6a^3(ab - 4b - 10)$$

Factor Trinomials when $a = 1$

Factoring Trinomials when $a = 1$

Make sure it's in standard form:

$$ax^2 + bx + c$$

Ask the question:

What multiplies to be " c ", but adds to be " b "?

When C is positive:

Signs in factors will be same $++$ or $--$

When C is negative:

Signs in factors are mixed $+ -$

1. $x^2 + 5x + 6$

$$(x+2)(x+3)$$

2. $x^2 - 5x - 6$

$$(x-6)(x+1)$$

3. $x^2 + 11x + 24$

$$(x+8)(x+3)$$

4. $x^2 + 5x - 24$

$$(x-3)(x+8)$$

5. $x^2 - 13x + 36$

$$(x-9)(x-4)$$

6. $x^2 - 4x - 12$

$$(x-6)(x+2)$$

7. $x^2 - 15x + 36$

$$(x-3)(x-12)$$

8. $x^2 + 4x - 21$

$$(x+7)(x-3)$$

9. $2x^2 + 12x + 16$

$$2(x^2 + 6x + 8)$$

$$2(x+4)(x+2)$$

10. $3x^2 - 9x - 120$

$$3(x^2 - 3x - 40)$$

$$3(x-8)(x+5)$$

Daily Check

Factoring when a=1 Daily Check

1. $r^2 - 11r + 28$

$$(r-4)(r-7)$$

$$\begin{array}{r} 28 \\ 1 \quad 28 \\ 2 \quad 14 \\ 4 \quad 7 \end{array}$$

2. $k^2 - 3k - 54$

$$(k-9)(k+6)$$

$$\begin{array}{r} 54 \\ + - \\ 1 \quad 54 \\ 2 \quad 27 \\ 3 \quad 18 \\ 6 \quad 9 \end{array}$$

3. $\frac{5r^2}{5} + \frac{35r}{5} + \frac{60}{5}$

$$5(r^2 + 7r + 12)$$
$$5(r+4)(r+3)$$

4. $x^2 - 9$

$$x^2 + 0x - 9$$
$$(x+3)(x-3)$$

Factor Trinomials

Factoring Trinomials when $a > 1$

Lets think about FOIL

1)

$$(2x+5)(3x+2)$$

$$6x^2 + 4x + 15x + 10$$

$$\boxed{6x^2 + 19x + 10}$$

2) $(3x+4)(x-6)$

$$3x^2 - 18x + 4x - 24$$

$$\boxed{3x^2 - 14x - 24}$$

3) $(7x+3)(x+5)$

$$7x^2 + 35x + 3x + 15$$

$$\boxed{7x^2 + 38x + 15}$$

4) $(2x+9)(3x-8)$

$$6x^2 - 16x + 27x - 72$$

$$\boxed{6x^2 + 11x - 72}$$

Scenario 1: a is prime

Make sure the trinomial is in Standard Form

$$2x^2 + 23x + 30$$

$$(2x + 3)(x + 10)$$

$$2x^2 + 20x + 3x + 30$$

30
1 30
2 15
3 10
5 6

$$3x^2 + 14x - 80$$

$$(3x - 10)(x + 8)$$

80
1 80
2 40
4 20
5 16
8 10

$$7x^2 - 38x + 40$$

$$(7x - 10)(x - 4)$$

40
1 40
2 20
4 10
5 8

$$3x^2 - x - 4$$

$$(3x - 4)(x + 1)$$

4
1 4
2 2

$$7x^2 - 28x - 10x + 40$$

$$5x^2 + 54x + 40$$

$$(5x + 4)(x + 10)$$

40
1 40
2 20
4 10
5 8

$$7x^2 - 46x - 21$$

$$(7x + 3)(x - 7)$$

21
1 21
3 7

$\begin{array}{r} 10x^2 + 49x + 18 \\ \underline{10x} \\ 52 \end{array}$ $(10x)$ $(5x + 2)(2x + 9)$ $\begin{array}{r} 18 \\ \underline{18} \\ 29 \\ \underline{36} \end{array}$	$\begin{array}{r} 6x^2 + 31x + 35 \\ \underline{6x} \\ 23 \end{array}$ $(6x^2 + 31x + 35)$ $(2x + 7)(3x + 5)$ $\begin{array}{r} 18 \\ \underline{18} \\ 29 \\ \underline{36} \end{array}$
$\begin{array}{r} 8x^2 + 30x + 27 \\ \underline{8x} \\ 24 \end{array}$ $(8x^2 + 30x + 27)$ $(4x + 9)(2x + 3)$ $\begin{array}{r} 27 \\ \underline{27} \\ 93 \\ \underline{93} \end{array}$	$\begin{array}{r} 9x^2 - 35x + 24 \\ \underline{9x} \\ 27 \end{array}$ $(9x^2 - 35x + 24)$ $(9x - 8)(x - 3)$ $\begin{array}{r} 24 \\ \underline{24} \\ 12 \\ \underline{12} \\ 38 \\ \underline{46} \end{array}$
$\begin{array}{r} 4x^2 - 21x - 18 \\ \underline{4x} \\ 22 \end{array}$ $(4x^2 - 21x - 18)$ $(4x + 3)(x - 6)$ $\begin{array}{r} 18 \\ \underline{18} \\ 29 \\ \underline{36} \end{array}$	$\begin{array}{r} 4r^2 - 15r - 4 \\ \underline{4r} \\ 22 \end{array}$ $(4r^2 - 15r - 4)$ $(4r + 1)(r - 4)$ $\begin{array}{r} 4 \\ \underline{4} \\ 14 \\ \underline{22} \end{array}$

a > 1 Daily Check

<p>1. $3r^2 + 20r + 32$</p> $\begin{array}{r} 12r \\ \underline{3r} \\ 8r \end{array}$ $(3r + 8)(r + 4)$ $\begin{array}{r} 32 \\ \underline{32} \\ 16 \\ \underline{16} \\ 48 \end{array}$	<p>2. $5a^2 + 16a - 16$</p> $\begin{array}{r} 20a \\ \underline{5a} \\ 11a \end{array}$ $(5a - 4)(a + 4)$ $\begin{array}{r} 16 \\ \underline{16} \\ 8 \\ \underline{8} \\ 44 \end{array}$
<p>3. $9v^2 + 59v + 30$</p> $\begin{array}{r} 54v \\ \underline{9v} \\ 50v \end{array}$ $(9v + 5)(v + 6)$ $\begin{array}{r} 30 \\ \underline{30} \\ 15 \\ \underline{15} \\ 10 \\ \underline{10} \\ 60 \end{array}$	<p>4. $10n^2 + 37n - 36$</p> $\begin{array}{r} 8n \\ \underline{10n} \\ 27n \end{array}$ $(2n + 9)(5n - 4)$ $\begin{array}{r} 36 \\ \underline{36} \\ 18 \\ \underline{18} \\ 12 \\ \underline{12} \\ 9 \\ \underline{9} \\ 66 \end{array}$

Special Cases

Difference of Two Squares Factoring Notes and Examples

What is a Perfect Square?	Can Variables be Perfect Squares?
$1^2 = 1$ $6^2 = 36$ $11^2 = 121$ $2^2 = 4$ $7^2 = 49$ $12^2 = 144$ $3^2 = 9$ $8^2 = 64$ $13^2 = 169$ $4^2 = 16$ $9^2 = 81$ $14^2 = 196$ $5^2 = 25$ $10^2 = 100$ $15^2 = 225$	$\sqrt{x^2} = x$ $\sqrt{x^4} = x^2$ $\sqrt{x^6} = x^3$ <i>Any even exponent gives you a perfect square.</i>
<p><u>Remember FOIL?</u> (Multiply out 1-4)</p> <p>1. $(x+8)(x+8)$</p> $x^2 + \underline{8x} + \underline{8x} + 64$ $\boxed{x^2 + 16x + 64} *$ <p><i>Perfect Square Trinomials</i> $\boxed{x^2 + 10x + 25} *$</p>	<p>2) $(x+5)(x+5)$</p> $x^2 + 5x + 5x + 25$ $\boxed{x^2 + 10x + 25} *$
<p>3) $(x+8)(x-8)$</p> $x^2 + \underline{8x} - \underline{8x} - 64$ $\boxed{x^2 - 64}$	<p>4) $(x-5)(x+5)$</p> $x^2 + \underline{5x} - \underline{5x} - 25$ $\boxed{x^2 - 25}$
<p><u>What's happening in #3 and #4?</u></p> <p>The "b" value cancels out.</p>	<p><u>Difference of Two Squares</u></p> $x^2 - 25$ $x^2 + 0x - 25$ $(x+5)(x-5)$
<p>5) $(x^2 - 9)$</p> $x^2 + 0x - 9$ $(x+3)(x-3)$	<p>6) $(x^2 - 49)$</p> $x^2 + 0x - 49$ $(x+7)(x-7)$

7) Always Check for a GCF First
 $(2x^2 - 32)$

$$2(x^2 - 16)$$

$$\boxed{2(x+4)(x-4)}$$

8) $(4x^2 - 36)$

$$4(x^2 - 9)$$

$$4(x+3)(x-3)$$

8) Always make sure its fully Simplified.

$$(x^4 - 1)$$

9) $(\frac{4}{25}x^4 - 64y^2)$

$$(\frac{2}{5}x^2 + 8y)(\frac{2}{5}x^2 - 8y)$$

Factor each using Difference of Two Squares. Always check for a GCF First.

1) $3v^2 - 12$ $3(v^2 - 4)$
 $3(v+2)(v-2)$

2) $36x^2 - 100$ $4(9x^2 - 25)$
 $4(3x+5)(3x-5)$

3) $100n^2 - 16$ $4(25n^2 - 4)$
 $4(5n+2)(5n-2)$

4) $16n^2 - 36$ $4(4n^2 - 9)$
 $4(2n+3)(2n-3)$

5) $45v^2 - 5$ $5(9v^2 - 1)$
 $5(3v+1)(3v-1)$

6) $25a^2 - 1$
 $(5a+1)(5a-1)$

7) $b^2 - 9$
 $(b+3)(b-3)$

8) $27x^2 - 75$
 $3(9x^2 - 25)$
 $3(3x+5)(3x-5)$

What happens if it looks like

9) $(9 - x^2)$
 $(3+x)(3-x)$

10) $(16y^2 - 9x^2)$
 $(4y+3x)(4y-3x)$

Solving Quadratic Equations by Factoring

Solving Quadratic Equations by Factoring

1. Put the equation in Standard form.
2. **Factor Completely!**
3. Set each factor equal to **zero** and Solve. * zero product property *
4. Check your answers!!!

Solve each equation.

1. $x^2 - x - 56 = 0$

$$(x-8)(x-7) = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x-8=0 \quad x-7=0 \\ \boxed{x=8} \quad \boxed{x=7} \end{array}$$

3. $n^2 - 11n + 10 = 0$

$$(n-10)(n-1) = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ n-10=0 \quad n-1=0 \\ \boxed{n=10} \quad \boxed{n=1} \end{array}$$

5. $2x^2 + x - 15 = 0$

$$\frac{15}{53}$$

$$(2x-5)(x+3) = 0$$

$$\begin{array}{l} 2x-5=0 \quad x+3=0 \\ \boxed{x=5/2} \quad \boxed{x=-3} \end{array}$$

* not in standard *

2. $8x + x^2 + 7 = 0$

$$x^2 + 8x + 7 = 0$$

$$(x+7)(x+1) = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+7=0 \quad x+1=0 \\ \boxed{x=-7} \quad \boxed{x=-1} \end{array}$$

4. $\frac{3x^2 + 9x - 54}{3} = 0$

$$x^2 + 3x - 18 = 0$$

$$(x-3)(x+6) = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x-3=0 \quad x+6=0 \\ \boxed{x=3} \quad \boxed{x=-6} \end{array}$$

6. $4x^2 + 21x - 5 = 0$

$$4x^2 + 21x + 5 = 0$$

$$(4x+1)(x+5) = 0$$

$$\begin{array}{l} 4x+1=0 \quad x+5=0 \\ \boxed{x=-1/4} \quad \boxed{x=-5} \end{array}$$

Solve by Factoring Warm up

$$\textcircled{1} x^2 + 4x - 21 = 0$$

$$(x+7)(x-3) = 0$$

$$\boxed{x = -7} \quad \boxed{x = 3}$$

$\textcircled{2}$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$\boxed{x = 5} \quad \boxed{x = 2}$$

$$\textcircled{3} 2m^2 + 9m = 0$$

$$m(2m+9) = 0$$

$$\boxed{m = 0}$$

$$2m+9 = 0$$

$$\boxed{m = -\frac{9}{2}}$$

Solving Quadratic Equations by Taking the Square Root

Solving Quadratic Equations by Completing the Square

1. Get the x^2 or the **binomial** squared by itself.
2. Take the **square root** of BOTH sides of the equal sign
3. Don't forget the \pm sign
4. Simplify
5. Check your answers!!!

Solve each equation.

1. $x^2 + 4 = 0$

$$x^2 = -4$$

$$x = \sqrt{-4}$$

no real solutions

2. $\frac{1}{2}x^2 + 3 = 12$

$$\frac{1}{2}x^2 = 9$$

$$\sqrt{x^2} = \sqrt{18}$$

$$x = \pm\sqrt{18}$$

$$x = 3\sqrt{2}$$

$$x = -3\sqrt{2}$$

3. $2x^2 - 10 = -x^2 - 1$

$$3x^2 = 9$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = +\sqrt{3} \quad x = -\sqrt{3}$$

4. $(x+4)^2 - 3 = 15$

$$(x+4)^2 = 18$$

$$x+4 = \pm 3\sqrt{2}$$

$$x+4 = 3\sqrt{2}$$

$$x = -4 + 3\sqrt{2}$$

$$x+4 = -3\sqrt{2}$$

$$x = -4 - 3\sqrt{2}$$

5. $\frac{5(x-4)^2}{5} = \frac{125}{5}$

$$(x-4)^2 = 25$$

$$x-4 = 5$$

$$x-4 = -5$$

$$x = 9$$

$$x = -1$$

6. $-9x^2 = 243$

$$x^2 = -27$$

$$x = \sqrt{-27}$$

no real solutions

Solving Quadratic Equations by Completing the Square

1. Rewrite so all terms containing x are on one side.
2. Find the number that completes the square on the left side of the equation. Add that number to both sides.
3. Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.
4. Take the square root of each side.
5. Solve for x .
6. Check your answers!!!

Solve each equation.

1. $x^2 - 10x - 54 = 0$

$$\begin{aligned} (x^2 - 10x + \underline{25}) &= 54 + \underline{25} \\ \sqrt{(x-5)^2} &= \sqrt{79} \\ (x-5) &= \pm\sqrt{79} \\ \boxed{x = 5 \pm \sqrt{79}} \end{aligned}$$

2. $x^2 - 18x + 77 = 0$

$$\begin{aligned} x^2 - 18x + \underline{81} &= -77 + \underline{81} \\ \sqrt{(x-9)^2} &= \sqrt{4} \\ x-9 &= \pm 2 \\ x-9=2 & \quad x-9=-2 \\ \boxed{x=11} & \quad \boxed{x=7} \end{aligned}$$

3. $x^2 + 20x - 73 = 0$

$$\begin{aligned} x^2 + 20x + \underline{100} &= 73 + \underline{100} \\ \sqrt{(x+10)^2} &= \sqrt{173} \\ x+10 &= \pm\sqrt{173} \\ \begin{matrix} -10 & -10 \end{matrix} \\ \boxed{x = -10 \pm \sqrt{173}} \end{aligned}$$

$\left(\frac{b}{2}\right)^2$

4. $x^2 + 6x - 72 = -8$

$$\begin{aligned} x^2 + 6x + \underline{9} &= 64 + \underline{9} \\ \sqrt{(x+3)^2} &= \sqrt{73} \\ x+3 &= \pm\sqrt{73} \\ \boxed{x = -3 \pm \sqrt{73}} \end{aligned}$$

5. $x^2 - 10x - 56 = 6$

$$\begin{aligned} x^2 - 10x + \underline{25} &= 62 + \underline{25} \\ \sqrt{(x-5)^2} &= \sqrt{87} \quad \sqrt{3 \cdot 29} \\ x-5 &= \pm\sqrt{87} \\ \begin{matrix} +5 & +5 \end{matrix} \\ \boxed{x = 5 \pm \sqrt{87}} \end{aligned}$$

6. $x^2 - 14x - 75 = 8$

$$\begin{aligned} x^2 - 14x + \underline{49} &= 83 + \underline{49} \\ \sqrt{(x-7)^2} &= \sqrt{132} \\ x-7 &= \pm 2\sqrt{33} \\ \boxed{x = 7 \pm 2\sqrt{33}} \end{aligned}$$

Quadratic Formula

must be in standard form

The formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

equation must be set = 0

The discriminant: $b^2 - 4ac$

if $(b^2 - 4ac) > 0$ then 2 real solutions

if $(b^2 - 4ac) < 0$ then 2 imaginary solutions (no real)

if $(b^2 - 4ac) = 0$ then 1 real solution

QUADRATIC FORMULA DAILY CHECK

$d=88$

1. $3x^2 - 2x = 7$

$$3x^2 - 2x - 7 = 0$$

$a=3$ $b=-2$ $c=-7$

$$x = \frac{2 \pm \sqrt{(2)^2 - 4(3)(-7)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + 84}}{6}$$

$$x = \frac{2 \pm \sqrt{88}}{6}$$

$$x = \frac{2 \pm 2\sqrt{22}}{6}$$

$d=105$

2. $4x^2 + 5x - 2 = 3$

$$4x^2 + 5x - 5 = 0$$

$$\frac{-5 \pm \sqrt{25 - 4(4)(-5)}}{8}$$

$$\frac{-5 \pm \sqrt{25 + 80}}{8}$$

$$x = \frac{-5 \pm \sqrt{105}}{8}$$

$$x = \frac{1 \pm \sqrt{22}}{3}$$