

# Unit 3: Similarity & Trig

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- ⑦ Special Right  $\triangle$  Rules (pink/Green)
- ⑧ Special Right Practice (yellow)

## Ratios in Similar Polygons

Fill in the blanks to complete each definition.

1. A similarity ratio is the ratio of the lengths of the corresponding sides of two similar polygons.

2. Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.

3. Figures that are similar have the same shape but not necessarily the same size.

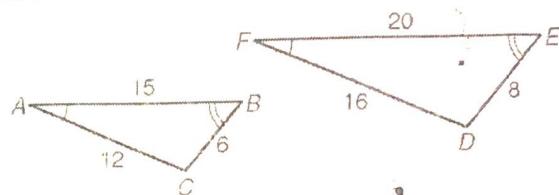
Use the figure for Exercises 4 and 5. The triangles are similar.

4. Name the pairs of congruent angles.

$$\angle A \cong \angle F$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle D$$



5. Write the corresponding side lengths in the proportion.

$$\frac{AB}{FE} = \frac{CB}{DE} = \frac{AC}{FD}$$

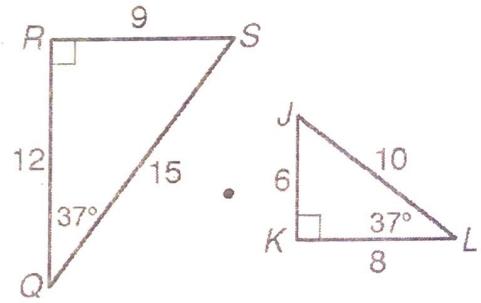
Use the figure to the right for Exercises 6 and 7. The triangles are similar.

6. Circle the correct similarity statement.

$$\Delta QRS \sim \Delta KJL \quad \boxed{\Delta RSQ \sim \Delta KJL} \quad \Delta QSR \sim \Delta LKJ$$

7. Write the corresponding side lengths in the proportion.

$$\frac{RS}{KJ} = \frac{RQ}{KL} = \frac{SQ}{JL}$$

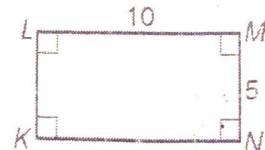
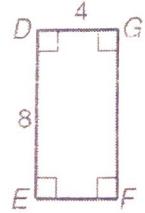


Use the figure to the right for Exercise 8.

8. Substitute numbers for the side lengths and reduce each ratio to simplest form.

$$\frac{DG}{MN} = \frac{4}{5} \quad \checkmark$$

$$\frac{DE}{LM} = \frac{8}{10} = \frac{4}{5} \quad \checkmark$$



## Scale Factor

Scale Factor – the ratio of corresponding sides

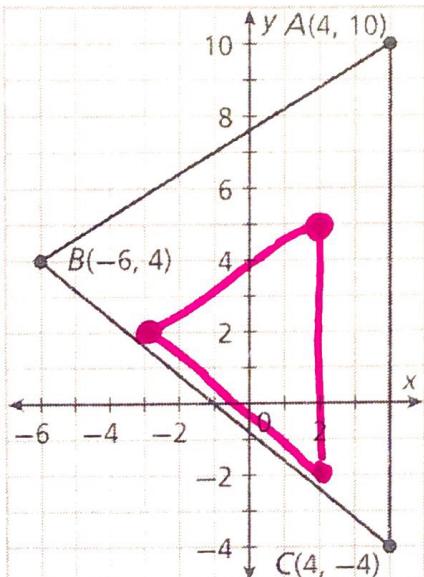
- When scale factor is greater than 1, the shape gets bigger and this is called an enlargement.
- When scale factor is less than 1, but greater than 0, the shape gets smaller and this is called a reduction.
- Formula: New Image  
Original Pre-Image

## Dilations

Apply the dilation  $D$  to the polygon with the given vertices. Name the coordinates of the image points. Identify and describe the transformation as an enlargement or reduction.

9.  $D(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

A(4, 10), B(-6, 4), and C(4, -4)



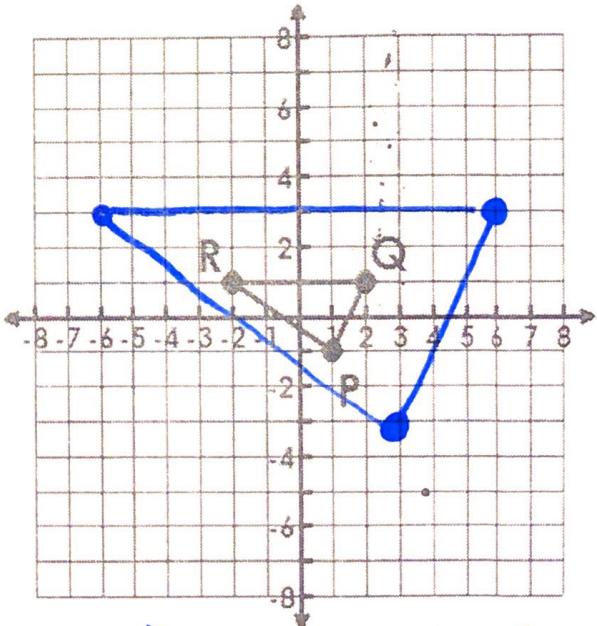
$A'$  (2, 5),  $B'$  (-3, 2), and  $C'$  (2, -2)

This shape is a/n Reduction.

The scale factor is  $\frac{1}{2}$ .

10.  $D(x, y) \rightarrow (3x, 3y)$

P(1, -1), Q(2, 1), R(-2, 1)



$P'$  (3, -3),  $Q'$  (6, 3), and  $R'$  (-6, 3)

This shape is a/n enlargement.

The scale factor is 3.

## Triangle Midsegment and Proportionality Theorem

Triangle Midsegment Theorem: The segment connecting the midpoints of two sides of the triangle is parallel to the third side and half the length of the third side.

Use  $\triangle ABC$ , where L, M, and N are midpoints of the sides.

$$1. \overline{LM} \parallel \overline{BC}$$

$$2. \overline{AB} \parallel \overline{MN}$$

$$3. \text{ If } AC = 20, \text{ then } LN = \underline{10}$$

$$4. \text{ If } MN = 7, \text{ then } AB = \underline{14}$$

$$5. \text{ If } NC = 9, \text{ then } LM = \underline{9}$$

$$6. \text{ If } \underline{\text{MID}} = 3x + 7, \text{ and } \underline{\text{Para}} = BC = 7x + 6, \text{ then } LM = \underline{31}$$

**2(midsegment) = parallel side**

$$2(3x+7) = 7x+6$$

$$\begin{aligned} 6x + 14 &= 7x + 6 \\ 18 &= x \end{aligned}$$

$$\begin{aligned} 3(8)+7 &= 31 \\ 24+7 &= 31 \end{aligned}$$

$$7. \text{ If } MN = x - 1, \text{ and } AB = 6x - 18, \text{ then } AB = \underline{6}$$

$$2(x-1) = 6x-18$$

$$2x-2 = 6x-18$$

$$x=4$$

$$\begin{aligned} 6(4)-18 &= 6 \\ 24-18 &= 6 \end{aligned}$$

$$16 = 4x$$

8. Find each measure. H, G, and I are all midpoints.

$$a) HI = \underline{9.1}$$

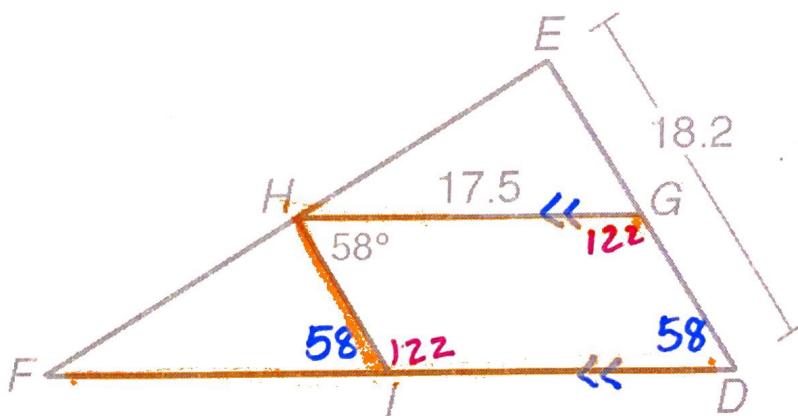
$$b) DF = \underline{35}$$

$$c) GE = \underline{9.1}$$

$$d) m\angle HIF = \underline{58}$$

$$e) m\angle HGD = \underline{122}$$

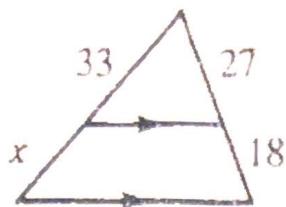
$$f) m\angle D = \underline{58}$$



Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

Find the value of x:

9.

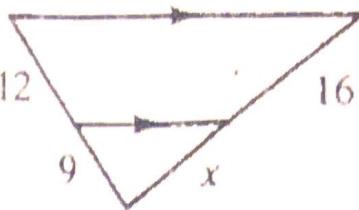


$$\frac{33}{x} = \frac{27}{18}$$

$$27x = 594$$

$$x = 22$$

10.

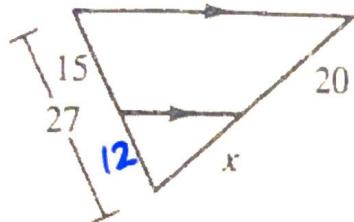


$$\frac{12}{9} = \frac{16}{x}$$

$$12x = 144$$

$$x = 12$$

11.



$$\frac{15}{12} = \frac{20}{x}$$

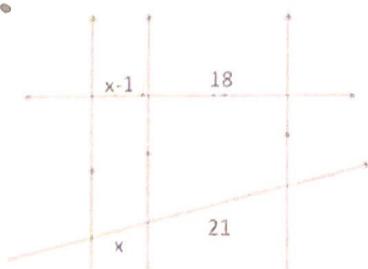
$$15x = 240$$

$$x = 16$$

$$27 - 15 = 12$$

This works for Parallel lines

cutting  
multiple  
transversals!



$$\frac{18}{x-1} = \frac{21}{x}$$

$$18x = 21x - 21$$

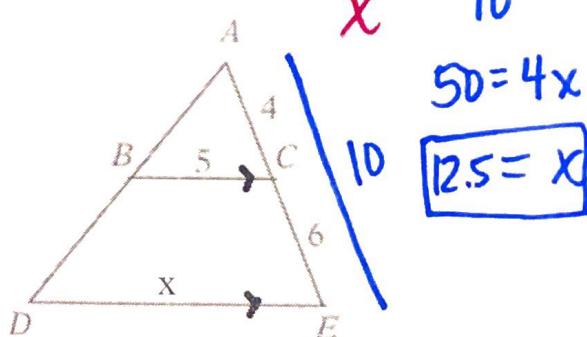
$$x = 7$$

$$21 = 3x$$

8

\*\*\*

13.



$$\frac{5}{x} = \frac{4}{10}$$

$$50 = 4x$$

$$125 = x$$

\*

$$\frac{6}{18} = \frac{9}{x}$$

$$16.2 = 6x$$

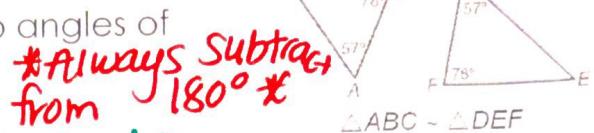
$$12.7 = x$$



### 3 Ways to Prove Triangles are Similar

#### AA~ Postulate:

If two angles of one triangle are congruent to two angles of another, then the triangles are similar.



$$\triangle ABC \sim \triangle DEF$$

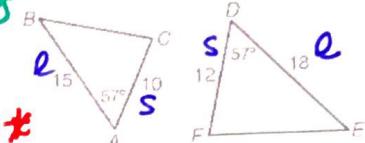
#### \* Look for parallel lines \*

\*vertical\*

#### SAS~ Postulate: Test scale factor of corresponding sides

If one angle of one triangle is congruent to the one angle of another triangle and the adjacent sides are proportional, then the triangles are similar.

\*included angle\*



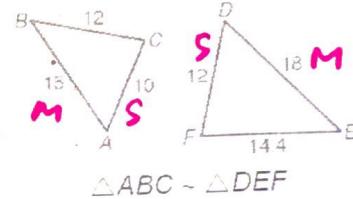
$$\triangle ABC \sim \triangle DEF$$

$$\frac{10}{12} = \frac{15}{18} \quad \frac{5}{6} \checkmark \quad .83 \checkmark \quad 180 = 180$$

#### SSS~ Postulate: Test scale factor...

If all three sides of one triangle are proportional to corresponding sides of another triangle, then the triangles are similar.

$$\frac{10}{12} \checkmark \quad \frac{15}{18} \checkmark \quad \frac{12}{14.4} \checkmark \quad .83 \checkmark$$

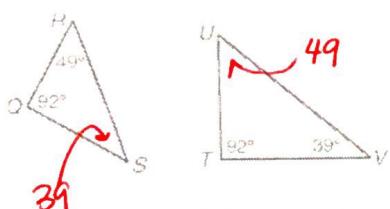


$$\triangle ABC \sim \triangle DEF$$

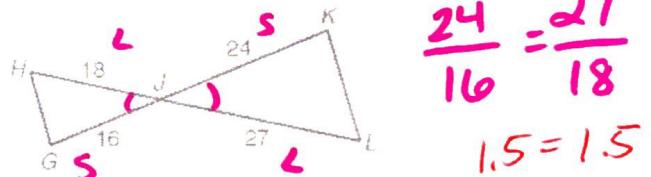
You can mark vertical angles and shared angles congruent!

Explain why the triangles are similar (SSS~, SAS~, or AA~) and write a similarity statement.

1)  $\triangle RQS \sim \triangle UTV$  by AA



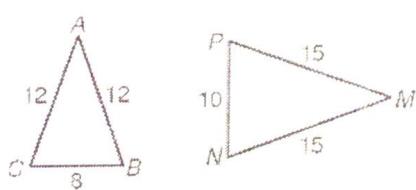
2)  $\triangle HGJ \sim \triangle LKJ$  by SAS



$$\frac{24}{16} = \frac{27}{18}$$

$1.5 = 1.5 \checkmark$

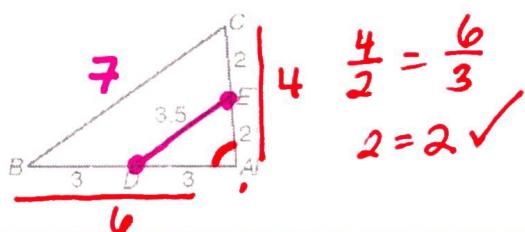
3)  $\triangle ABC \sim \triangle MNP$  by SSS



$$\frac{12}{15} = \frac{8}{10} = \frac{12}{15}$$

$$\frac{4}{5} = \frac{4}{5}$$

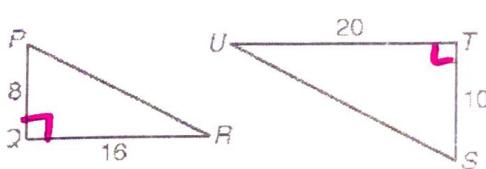
4)  $\triangle ADE \sim \triangle ABC$  by SAS



$$\frac{4}{3} = \frac{6}{4}$$

$2 = 2 \checkmark$

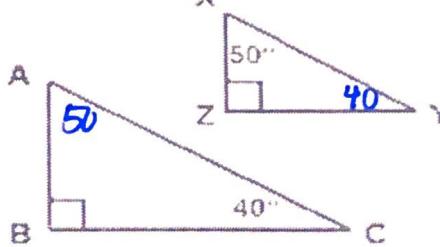
5)  $\triangle QPR \sim \triangle TSU$  by SAS



$$\frac{16}{20} = \frac{8}{10}$$

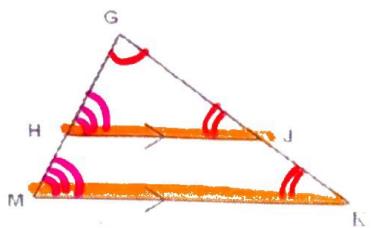
$$\frac{4}{5} = \frac{4}{5}$$

6)  $\triangle ABC \sim \triangle XZY$  by AA



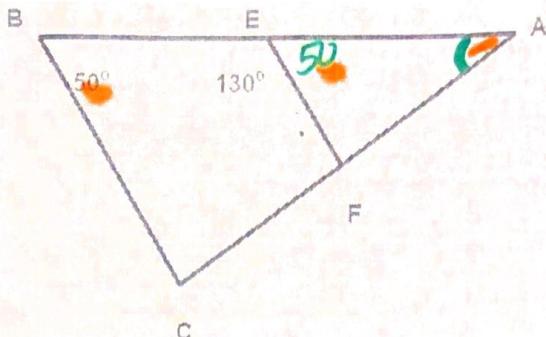
## GSE Geometry

7)  $\triangle GHJ \sim \triangle GMK$  by AA



## 3 - Similarity &amp; Right Triangles

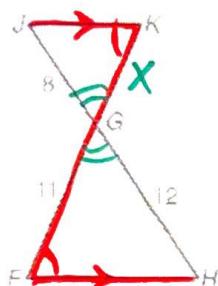
8)  $\triangle AEF \sim \triangle ABC$  by AA



\* Do they share any angles?

Explain why the triangles are similar (SSS~, SAS~, or AA~) and find each length.

9) Similar by AA and  $GK = 7.\bar{3}$

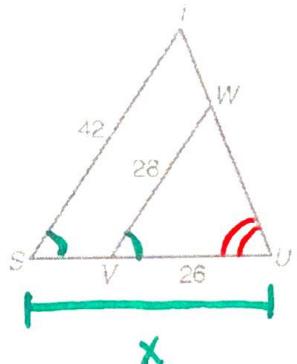


$$\frac{8}{12} = \frac{x}{11}$$

$$12x = 88$$

$$x = 7.\bar{3}$$

10) Similar by AA and  $SU = 39$

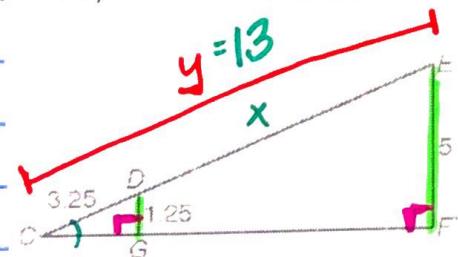


$$\frac{42}{28} = \frac{x}{26}$$

$$28x = 1092$$

$$x = 39$$

11) Similar by AA and  $DE = 9.75$



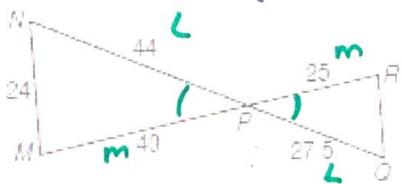
$$\frac{13}{3.25} = \frac{3.25}{y}$$

$$\frac{1.25}{5} = \frac{3.25}{y}$$

$$1.25y = 16.25$$

$$y = 13$$

12) Similar by SAS and  $RQ = 15$



$$\frac{40}{25} = \frac{44}{27.5}$$

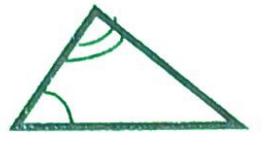
$$1.6 = 1.6$$

$$\frac{40}{25} = \frac{24}{x}$$

$$x = 15$$

# AA

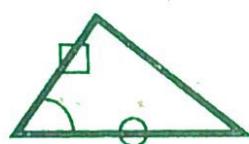
## Similarity



If two angles of one triangle are *congruent* to two angles of another triangle, then the triangles are *similar*.

# SAS

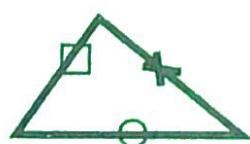
## Similarity



In two triangles, if a pair of corresponding angles is *congruent* and the sides including the angle are *proportional*, then the triangles are *similar*.

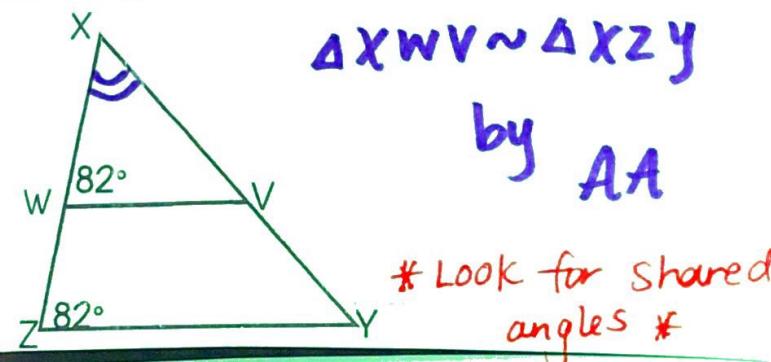
# SSS

## Similarity

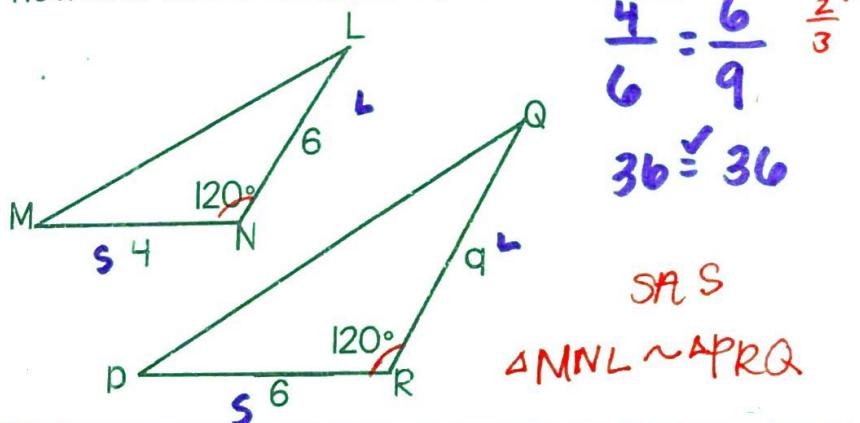


If all three pairs of corresponding sides of two triangles are *proportional*, then the two triangles are *similar*.

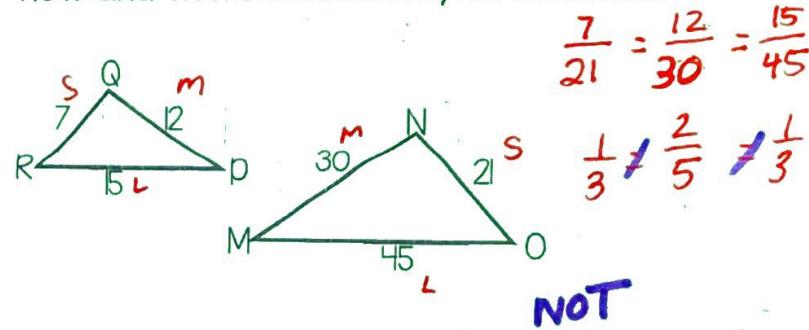
EX 1: Are the two triangles similar? If so, state how and write a similarity statement.



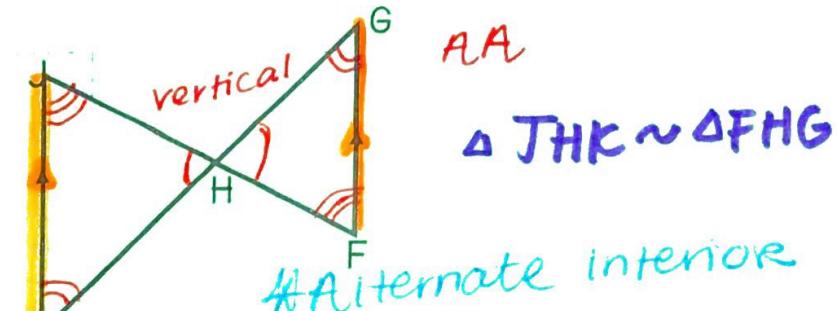
EX 3: Are the two triangles similar? If so, state how and write a similarity statement.



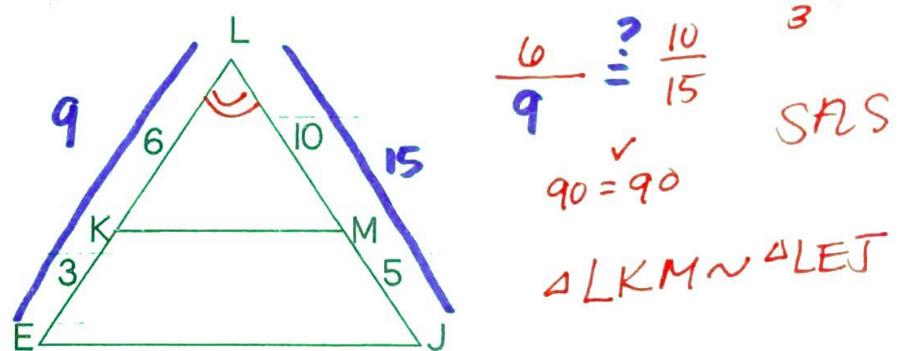
EX 5: Are the two triangles similar? If so, state how and write a similarity statement.



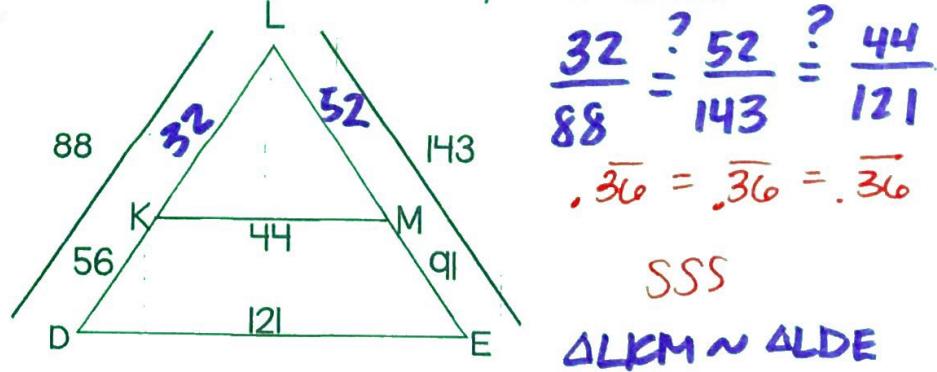
EX 2: Are the two triangles similar? If so, state how and write a similarity statement.



EX 4: Are the two triangles similar? If so, state how and write a similarity statement.



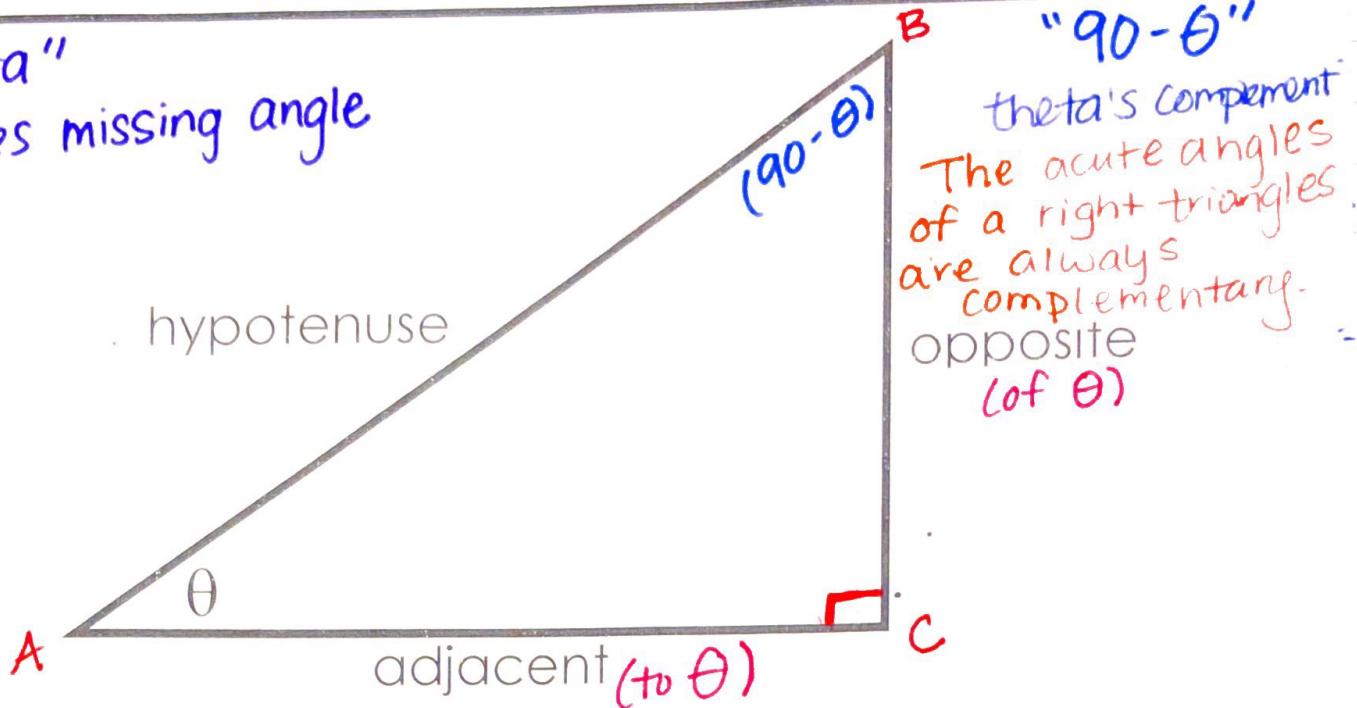
EX 6: Are the two triangles similar? If so, state how and write a similarity statement.



# Trigonometry Ratios

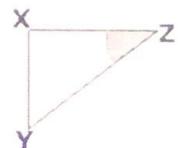
$\theta$  "theta"

denotes missing angle



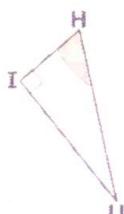
Sine	Cosine	Tangent
$\sin(\theta) = \frac{3}{5}$	$\cos(\theta) = \frac{4}{5}$	$\tan(\theta) = \frac{3}{4}$
SOH	CAH	TOA
$S = \frac{O}{H}$	$C = \frac{A}{H}$	$T = \frac{O}{A}$

1. Identify the side that is opposite  $\angle Z$   $\overline{xy}$



2. Identify the side that is adjacent to  $\angle Z$   $\overline{xz}$

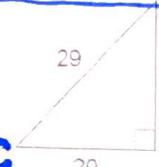
3. Identify the side that is opposite  $\angle H$   $\overline{iu}$



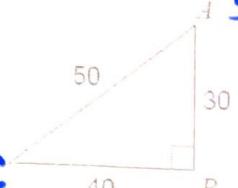
4. Identify the side that is adjacent to  $\angle H$   $\overline{ih}$

Find the value of each trigonometric ratio. Express your answer as a fraction in lowest terms.

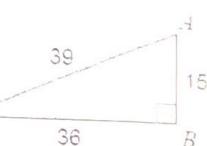
5.  $\sin C = \frac{21}{29}$



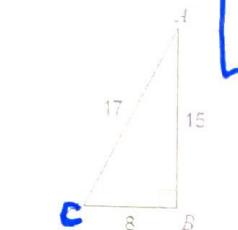
6.  $\sin C = \frac{30}{50} = \frac{3}{5}$



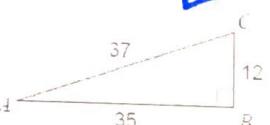
7.  $\cos C = \frac{36}{39} = \frac{12}{13}$



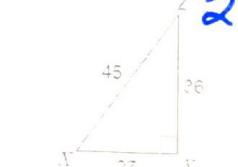
8.  $\cos C = \frac{8}{17}$



9.  $\tan A = \frac{12}{35}$



10.  $\tan X = \frac{36}{27} = \frac{4}{3}$



For each of the following find the trigonometric ratio.

11.  $\sin A$

$\frac{15}{17}$

12.  $\cos A$

$\frac{8}{17}$

13.  $\tan A$

$\frac{15}{8}$

14.  $\sin B$

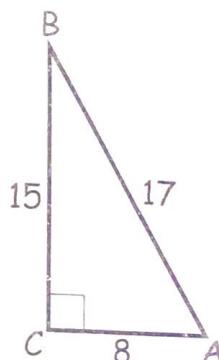
$\frac{8}{17}$

15.  $\cos B$

$\frac{15}{17}$

16.  $\tan B$

$\frac{8}{15}$



\* The tangents of complementary angles will be reciprocals.

How do your answers in #11-13 compare to those in #14-16?

The sine of an angle is always equal to the cosine of its complement.

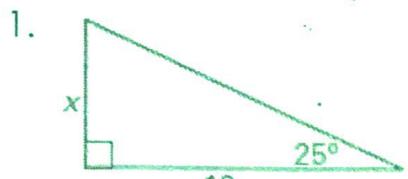
# missing Sides & Missing Angles

(use Trig)

(use inverse trig)

SOH CAH TOA

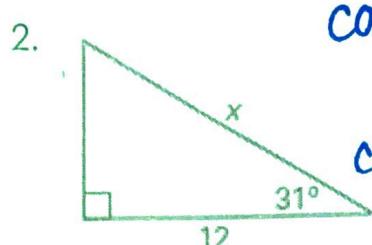
Using Trig Ratios to find Missing Sides or Angles



$$\tan(25) = \frac{x}{12}$$

$$12 \cdot \tan(25) = x$$

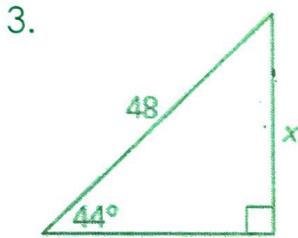
$$x = 5.6$$



$$\cos(31) = \frac{12}{x}$$

$$\frac{12}{\cos(31)} = x$$

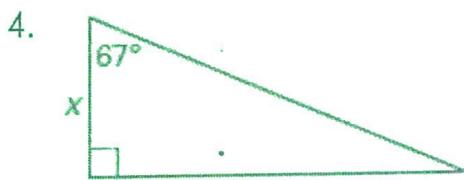
$$14 = x$$



$$\sin(44) = \frac{x}{48}$$

$$48 \cdot \sin(44) = x$$

$$x = 33.3$$

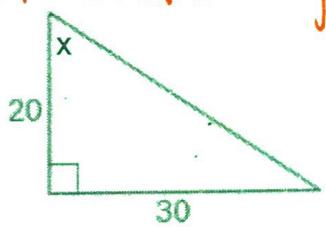


$$\tan(67) = \frac{18}{x}$$

$$x = \frac{18}{\tan(67)}$$

$$x = 7.6$$

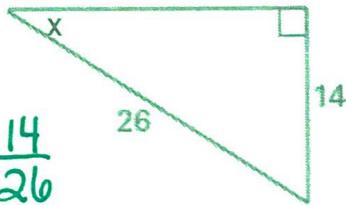
\* To Find an angle use the inverse \*



$$\tan(x) = \frac{30}{20}$$

$$\tan^{-1}(30/20) = x$$

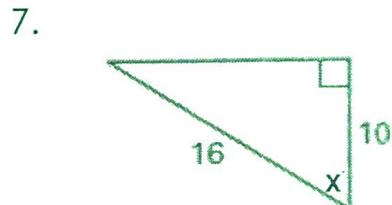
$$x = 56.3$$



$$\sin(x) = \frac{14}{26}$$

$$\sin^{-1}\left(\frac{14}{26}\right) = x$$

$$x = 32.6^\circ$$



$$\cos(x) = \frac{10}{16}$$

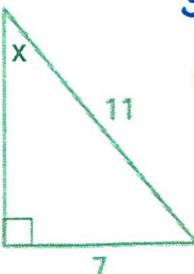
$$\cos^{-1}\left(\frac{10}{16}\right) = x$$

$$x = 51.3$$

\* \*

\*

\*



$$\sin(x) = \frac{7}{11}$$

$$\sin^{-1}\left(\frac{7}{11}\right) = x$$

$$x = 39.5^\circ$$

9.

$$\tan^{-1}\left(\frac{16}{30}\right) = x$$

$$28.1 = x$$

\*

10.

$$\tan^{-1}\left(\frac{11}{15}\right) = x$$

$$53.7 = x$$

\*

11.

$$\tan(38) = \frac{x}{20}$$

$$20 \tan 38 = x$$

$$15.6 = x$$

12.

$$\sin(42) = \frac{24}{b}$$

$$24 / \sin(42) = b$$

$$35.9 = b$$

13.

$$\cos(33) = \frac{17}{r}$$

$$\frac{17}{\cos(33)} = r$$

$$20.3 = r$$

14.

\*

$$\cos(x) = \frac{9}{22}$$

$$\cos^{-1}\left(\frac{9}{22}\right) = x$$

$$x = 65.9$$

15.

$$\tan^{-1}\left(\frac{16}{20}\right) = x$$

$$38.7 = x$$

\*

16.

$$\tan(43) = \frac{x}{24}$$

$$24 \cdot \tan(43) = x$$

$$22.4 = x$$

17.

$$\cos(61) = \frac{50}{d}$$

$$50 / \cos(61) = d$$

$$d = 103.13$$

18.

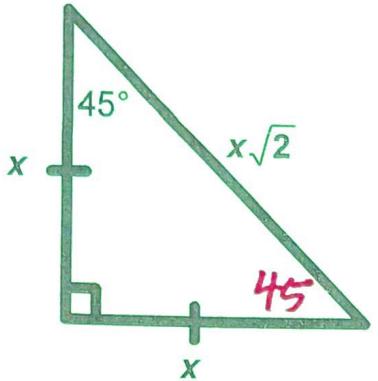
\*

$$\tan^{-1}\left(\frac{17}{12}\right) = x$$

$$x = 54.8$$

# Special Right

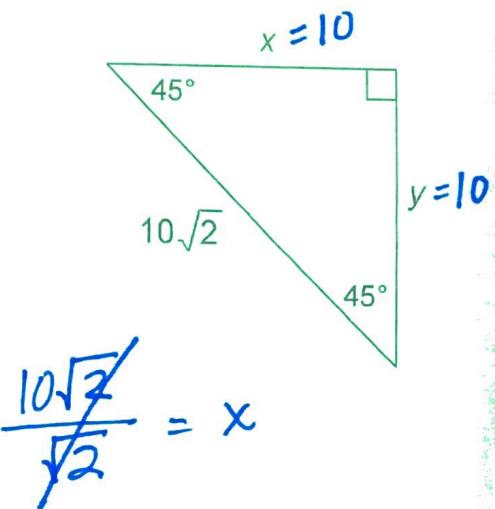
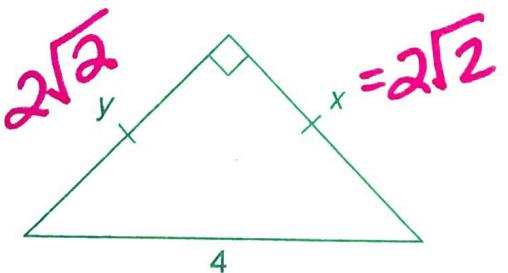
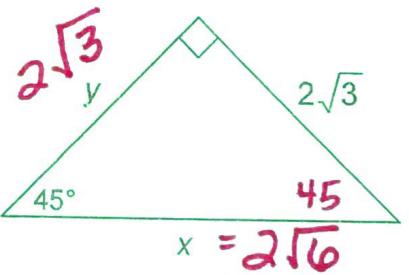
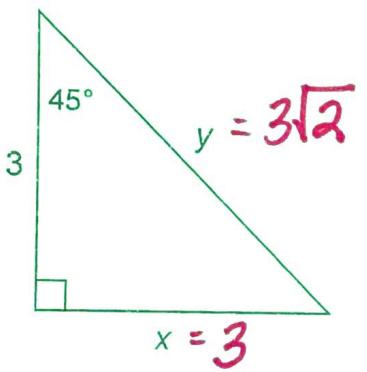
## 45°-45°-90° TRIANGLES



$$\text{leg} = \text{leg}$$

$$\text{hyp} = \sqrt{2} \cdot \text{leg}$$

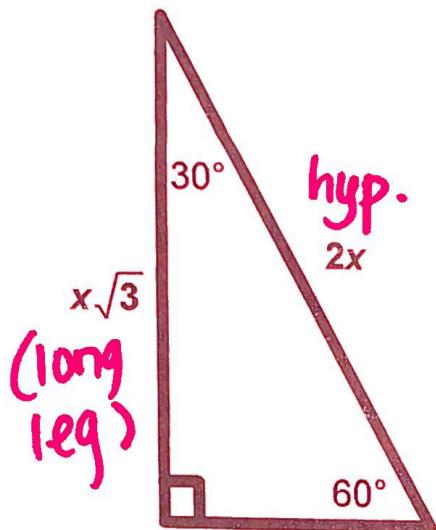
$$\text{leg} = \frac{\text{hyp}}{\sqrt{2}}$$



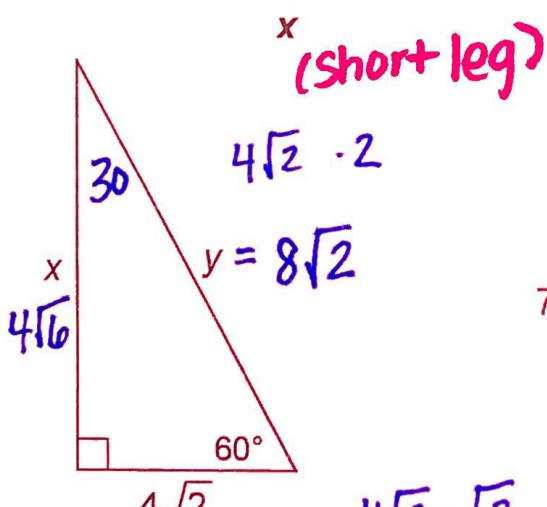
# Triangles

NON  
calculator

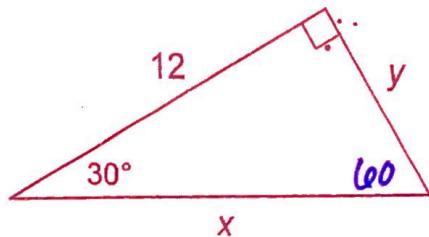
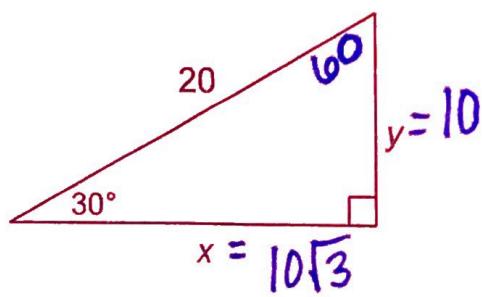
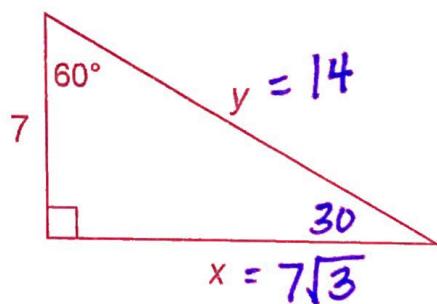
## 30°-60°-90° Triangles



$$\begin{aligned} \text{hyp} &= 2 \cdot \text{Short} \\ \text{leg} &= \sqrt{3} \cdot \text{Short} \\ \text{Short} &= \frac{\text{hyp}}{2} \\ \text{Short} &= \frac{\text{long}}{\sqrt{3}} \end{aligned}$$

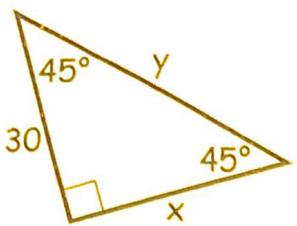


$$x = 4\sqrt{6}$$

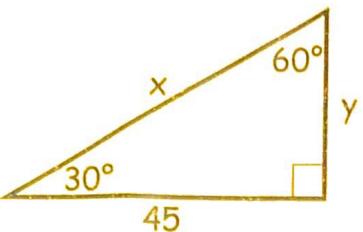


# more Special Right

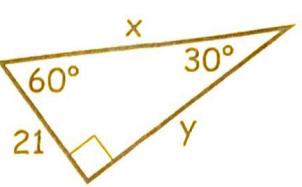
1.  $x = 30$ ,  $y = 30\sqrt{2}$



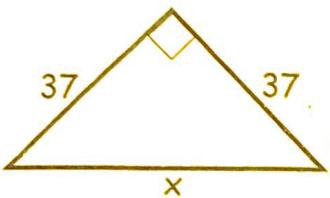
2.  $x = 30\sqrt{3}$ ,  $y = 15\sqrt{3}$



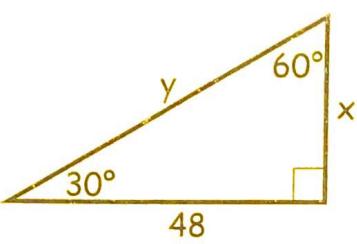
3.  $x = 42$ ,  $y = 21\sqrt{3}$



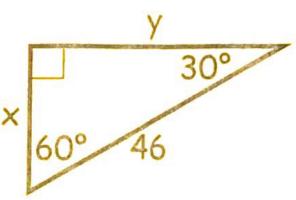
4.  $x = 37\sqrt{2}$



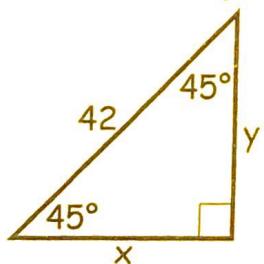
5.  $x = 16\sqrt{3}$ ,  $y = 32\sqrt{3}$



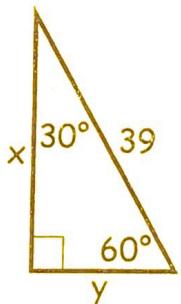
6.  $x = 23$ ,  $y = 23\sqrt{3}$



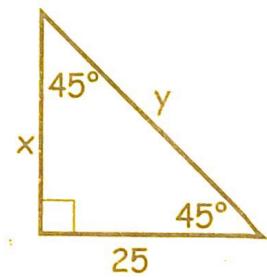
7.  $x = 21\sqrt{2}$ ,  $y = 21\sqrt{2}$



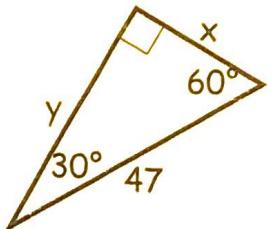
8.  $x = 19.5\sqrt{3}$ ,  $y = 19.5$



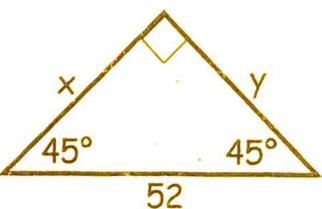
9.  $x = 25$ ,  $y = 25\sqrt{2}$



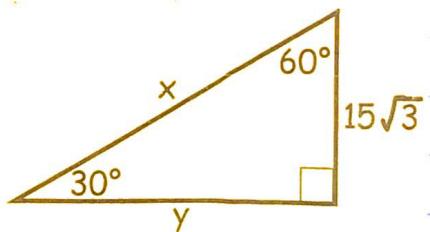
10.  $x = 23.5$ ,  $y = 23.5\sqrt{2}$



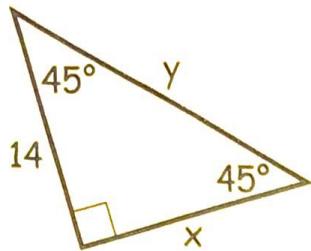
11.  $x = 26\sqrt{2}$ ,  $y = 26\sqrt{2}$



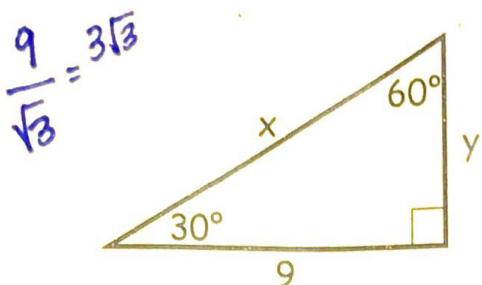
12.  $x = 30\sqrt{3}$ ,  $y = 45$



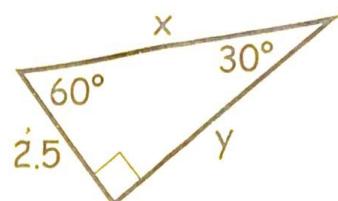
13.  $x = \underline{14}$ ,  $y = \underline{14\sqrt{2}}$



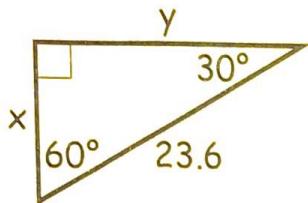
14.  $x = \underline{6\sqrt{3}}$ ,  $y = \underline{3\sqrt{3}}$



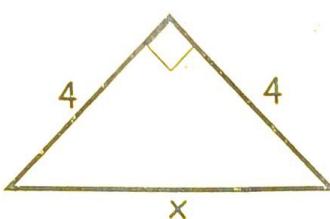
15.  $x = \underline{5}$ ,  $y = \underline{2.5\sqrt{3}}$



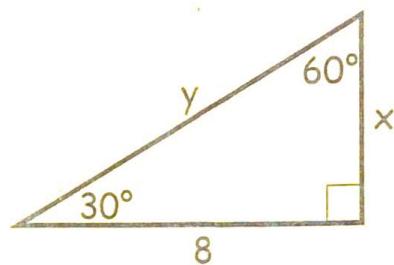
16.  $x = \underline{11.8}$ ,  $y = \underline{11.8\sqrt{3}}$



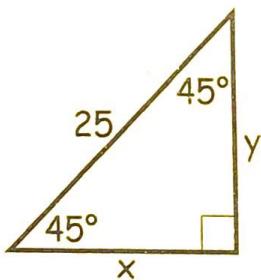
17.  $x = \underline{4\sqrt{2}}$



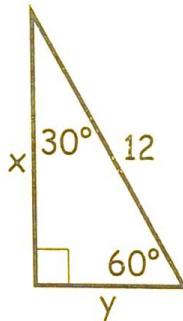
18.  $x = \underline{\frac{8\sqrt{3}}{3}}$ ,  $y = \underline{\frac{16\sqrt{3}}{3}}$



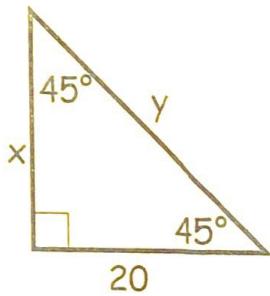
19.  $x = \underline{\frac{25\sqrt{2}}{2}}$ ,  $y = \underline{\frac{25\sqrt{2}}{2}}$



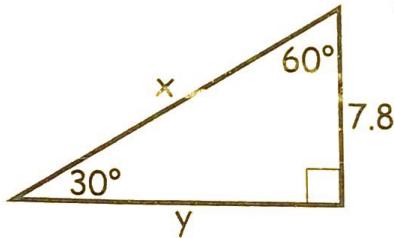
20.  $x = \underline{6\sqrt{3}}$ ,  $y = \underline{6}$



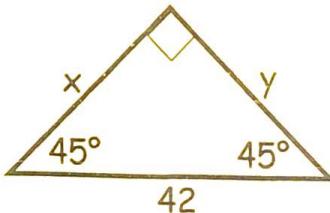
21.  $x = \underline{20}$ ,  $y = \underline{20\sqrt{2}}$



22.  $x = \underline{15.6}$ ,  $y = \underline{7.8\sqrt{3}}$



23.  $x = \underline{21\sqrt{2}}$ ,  $y = \underline{21\sqrt{2}}$



24.  $x = \underline{15}$ ,  $y = \underline{15\sqrt{3}}$

