Absolute Value & Piecewise Functions Final Review

Use the piecewise functions to answer the questions:

$$g(x) = \begin{cases} (x+3)^2, & x < -3 \\ 3 + \sqrt{x}, & -3 \le x < 2 \\ 2x - 4, & x \ge 2 \end{cases} \qquad f(x) = \begin{cases} (x-3)^2, & x < 4 \\ -2x + 4, & x = 4 \\ 1 + \sqrt{x}, & x > 4 \end{cases} \qquad s(x) = \begin{cases} (x-1)^2, & x \le 1 \\ (x-4)^4, & x \ge 3 \end{cases}$$

Evaluate:

1. $g(1) =$ **4**

2. $f(4) =$ **-4**

3. $s(2) =$ **none**

4. $s(4) =$ **0**

5. $g(-6) =$ **9**

6. $f(1) =$ **4**

7. $s(-6) =$ **49**

8. $g(-4) =$ **1**

Find the domain of each:

9. $g(x)$ $(-\infty, -3) \cup [0, \infty)$

10. $f(x)$ $(-\infty, \infty)$

11. $s(x)$ $(-\infty, 1] \cup [3, \infty)$

Determine if the function is continuous. Justify your answer.

12. $g(x) =$ no, gaps in domain.

13. $f(x)$ no, there are jumps.

14. $s(x)$ no, gaps in domain.

Graph the function:

15. $g(x)$

$f(x)$

s(x)

16.
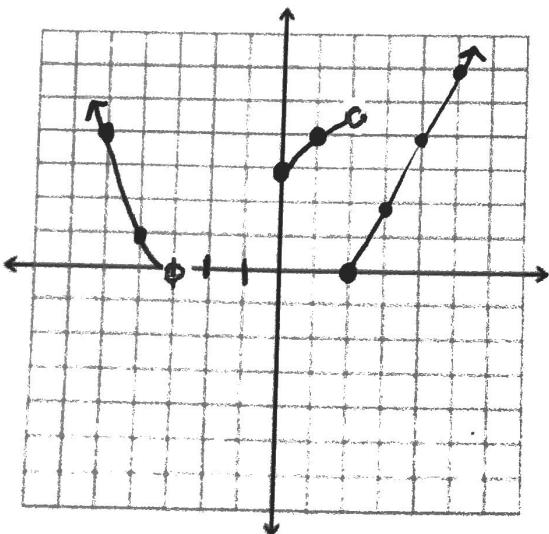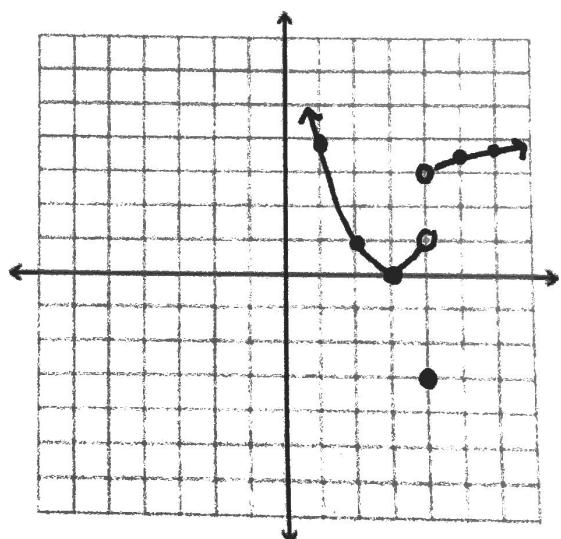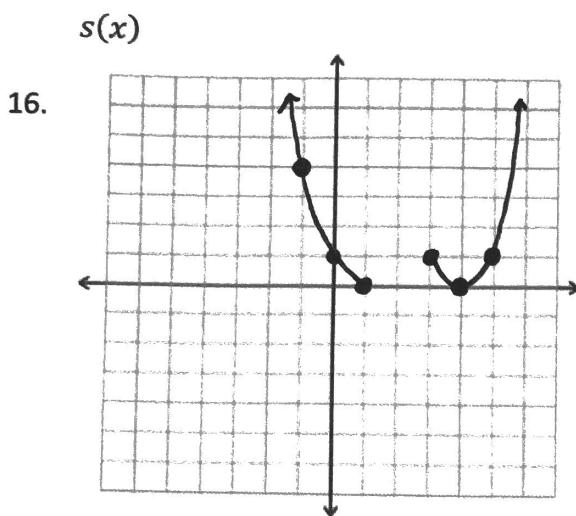
Graph the function. Find the domain and range. Then write as a piecewise function.

17. $f(x) = 2|x - 1| + 2$
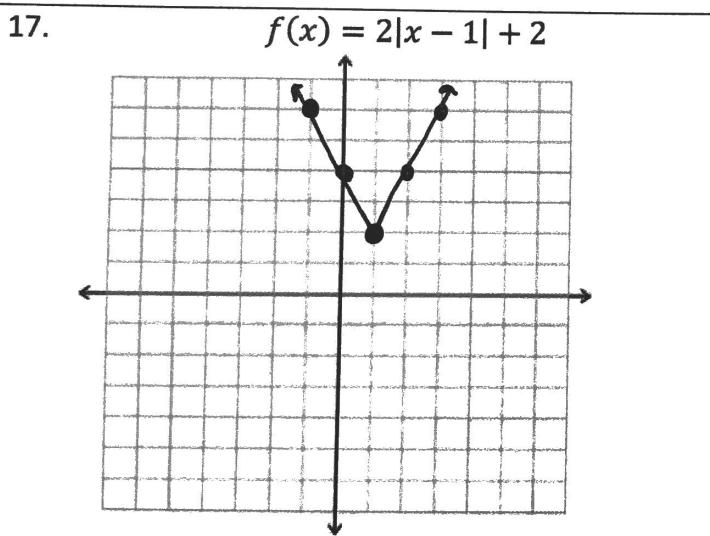
Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

$-2(x-1)+2$
$-2x+2+2$

Piecewise:

$$f(x) = \begin{cases} 2x & , x \geq 1 \\ -2x+4 & , x < 1 \end{cases}$$

18. $g(x) = -|x - 2| + 3$
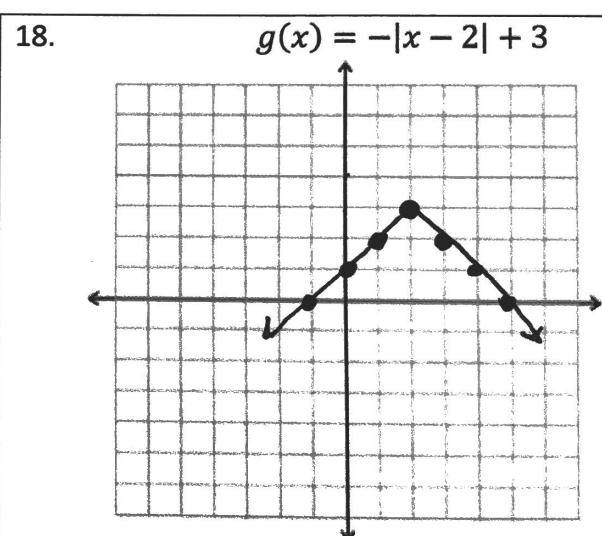
Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

Piecewise:

$$f(x) = \begin{cases} -x+5 & , x \geq 2 \\ x+1 & , x < 2 \end{cases}$$

19. Write $f(x)$ if the function is the graph of $|x|$ that has been reflected across the x axis, vertically compressed by $\frac{2}{5}$, and translated up 2 units.

$$y = -\frac{2}{5}|x| + 2$$

$|x - \text{midpoint}| \underset{\text{or}}{\overset{\geq}{\leq}} \text{distance}$

20. Write the absolute value expression representing all values that are at least 12 units from -2.

midpoint

$$|x + 2| \geq 12$$

distance

21. Write the absolute value function representing all values that are at most 2 units from 0.

$|x - 0| \leftarrow$ midpoint

$$|x| \leq 2$$

distance

22. The amount of Social Security tax you pay, part of your Federal Insurance Contributions Act (FICA) deductions, depends on your annual income. As of 1999 you pay 6.2% of your income if it is less than \$72,600. If your income is at least \$72,600, you pay a fixed amount of \$4501.20.

a. Write a piecewise function that gives the Social Security tax.

$$f(x) = \begin{cases} .062x & , x < 72,600 \\ \$4501.2 & , x \geq 72,600 \end{cases}$$

b. How much Social Security tax do you pay if you make \$30,000 per year?

$1860

23. During a nine hour snowstorm it snows at a rate of 1 inch per hour for the first two hours, at a of rate of 2 inches per hour for the next six hours, and at a rate of 1 inch per hour for the final hour.

a. Write a piecewise function that gives the depth of the snow during the snowstorm.

$$f(x) = \begin{cases} x & , x \leq 2 \\ 2x - 2 & , 2 < x \leq 8 \\ x + 6 & , 8 < x \leq 9 \end{cases}$$

b. How many inches of snow accumulated from the storm?

15

hr	Snow
1	1
2	2
3	4
4	6
5	8
6	10
7	12
8	14
9	15

Solve the absolute value equation/inequality.

24. $|3x + 12| + 7 = 7$ $\boxed{x = -4}$

$|3x + 12| = 0$

$3x = -12$

$x = 4$

Check

$|3(-4) + 12| + 7 = 7$

$|-12 + 12| + 7 = 7$

$0 + 7 = 7$ ✓

25. $|3x - 7| + 7 = 9$ $\boxed{x = 3 \quad x = 5/3}$

$|3x - 7| = 2$

$3x - 7 = 2$ $3x - 7 = -2$

$3x = 9$ $3x = 5$

$x = 3$ $x = 5/3$

check
3

26. $|x - 4| \geq 0$ $\boxed{\text{all real}}$

$x - 4 = 0$

$x = 4$

T   T   T

0   4   5

27. $-3 + |x + 1| \leq -3$ $\boxed{x = -1}$

$|x + 1| \leq 0$

$x = -1$

F   T   F

-2   -1   0
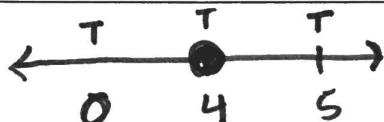
28. $2|x - 1| - 4 \geq 2$ $(-\infty, -2] \cup [4, \infty)$

$2|x-1| \geq 6$

$|x-1| \geq 3$

$x-1 = 3$ $x-1 = -3$

$x = 4$ $x = -2$

T   F   T
-2   0   4

29. $|2 - x| < 8$ $(-6, 10)$

$2 - x = 8$ $2 - x = -8$

$-x = 6$ $-x = -10$

$x = -6$ $x = 10$

-6   6   10
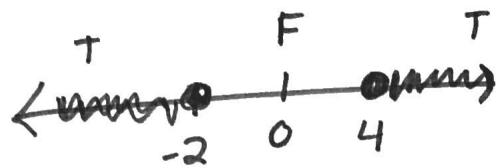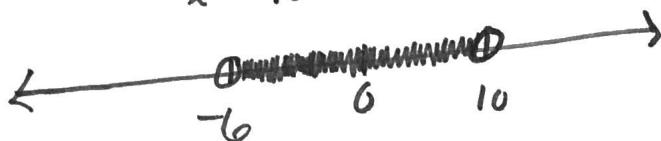
30. $|x + 6| > 0$ $(-\infty, -6) \cup (-6, \infty$

$x = -6$

31. $|x^2 + 5x + 4| = 0$ $x = -1 \mid x = -4$